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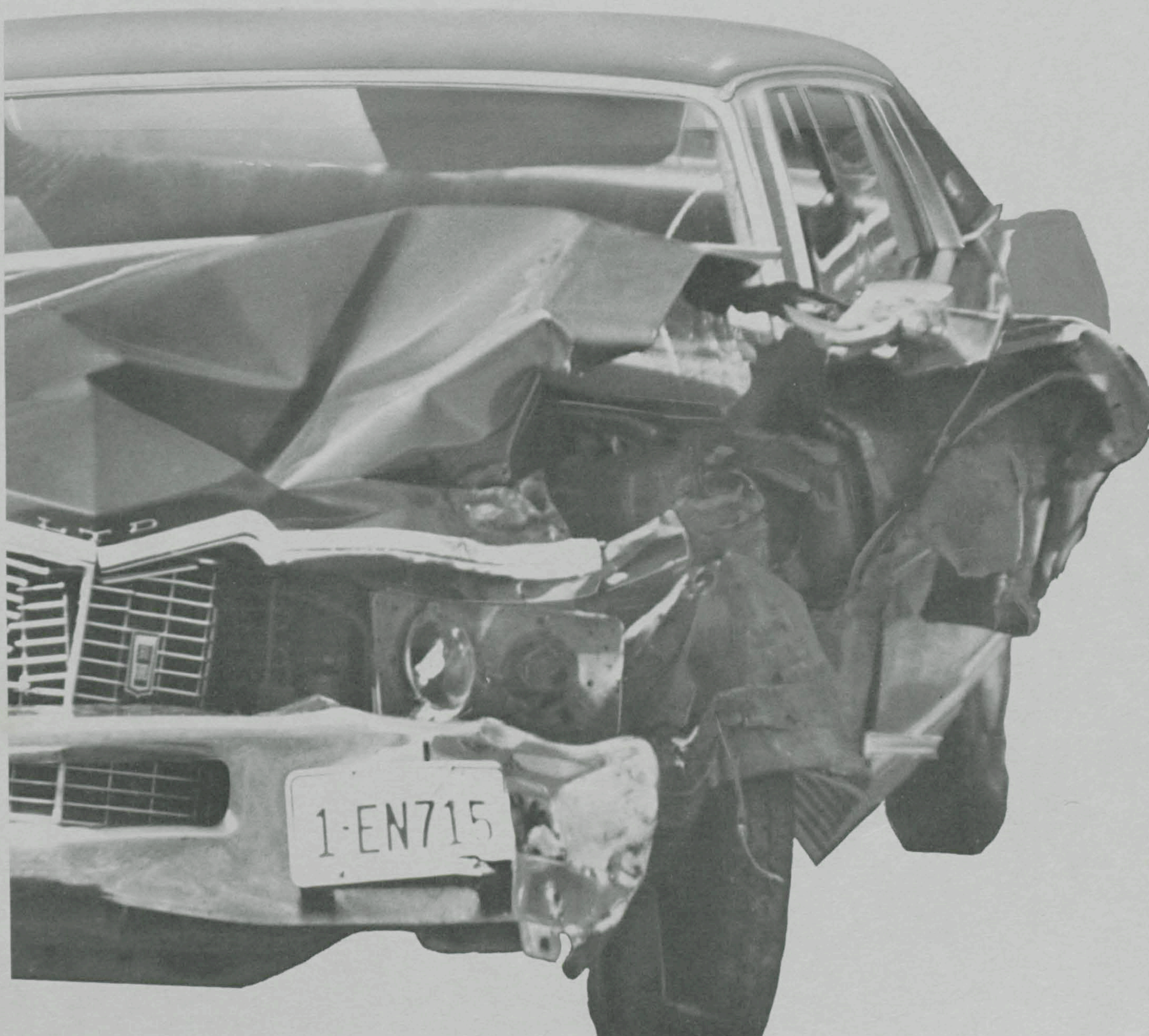

AUTOMOBILE COLLISIONS

Momentum and Energy

AUTOMOBILE COLLISIONS

A Module on Momentum and Energy

SUNY at Binghamton

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The Physics of Technology modules were produced by the Tech Physics Project, which was funded by grants from the National Science Foundation. The work was coordinated by the American Institute of Physics. In the early planning stages, the Tech Physics Project received a grant for exploratory work from the Exxon Educational Foundation.

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The authors wish to express their appreciation for the help of many people in bringing this module to final form. The criticisms of various reviewers and the cooperation of field-test teachers have been most helpful. Several members of the staff of the State University of New York at Binghamton also deserve special recognition for their contribution. They are Felix K. Chen, Duane A. Damiano, Eugene C. Deci, Barry R. DeGraff, Max Eibert, Steve I. Finette, Chung-Yu Fu, Jeffrey W. Gray, Leonard R. Kelly, Stevan H. Leiden, Debra M. Simes, Laurie A. Sperling, William Standish, and Howard M. Zendle.

Cover photograph taken by Dennis Purdy.

Automobile Collisions

ISBN 0-07-001726-3

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Automobile Collisions

INTRODUCTION

Many principles of physics are involved when automobiles collide. However, when the phenomenon you are studying is complicated, as automobile collisions are, it is wise to seek some quantity that stays the same no matter how drastically or how rapidly other quantities change. *Momentum* is such a quantity. In the absence of external forces, the total momentum of two colliding cars remains constant throughout the collision even though the speed of each car is abruptly changed, and even though large destructive internal forces cause enormous amounts of damage. When you have completed this module, you will be able to predict the results of simple collisions—not only those involving automobiles, but also collisions of billiard balls, atoms, and other objects whose masses, velocities, and energies are known.

There *are* other factors involved in analyzing the damage that results to both autos and passengers during a collision. In this module you will learn why cars experience less damage if they collide "off center," if they strike water-filled barriers rather than metal barriers, if they lift off the ground during collision, and if they strike highly elastic barriers that cause the cars to rebound. You will also gain some insight into methods of improving passenger safety. For example, why is a seat belt important and how should it be designed? Why are inflatable bags and ejector systems being considered and what are the unsolved problems that have prevented the adoption of these devices?

The physics studied in this module is basic to many other areas of science and technology in addition to automobile behavior. We hope you enjoy your study of this material and that it increases your ability to deal with phenomena that involve motion and collisions.

LEARNING GOALS FOR SECTION A

When you have completed your study of Section A of this module, you should be able to do the things in the following list. You will probably find it helpful to refer to it for review.

1. Define the following technical terms both in words and, if appropriate, in mathematical language: average velocity, uniform motion, acceleration, uniformly accelerated motion, mass, force, center of mass, lever arm, torque, kinetic energy, gravitational potential energy, elastic potential energy, and heat.

2. Given position versus time data for an object undergoing uniformly accelerated motion, determine the acceleration by using measurements made on appropriate graphs you plot.

3. Knowing the mass of an object and the forces acting on it, calculate its acceleration.

4. Given a rigid object that you can lift, such as a board or a meter stick, determine the location of the center of mass.

5. Knowing all the forces on an object and their points of application, calculate the torques produced by these forces. Determine from the sum of these torques whether or not the object is in static equilibrium.

6. If a body is in equilibrium, use the fact that the sum of the applied forces is zero to find any unknown force.

7. If a body is in equilibrium, use the fact that the sum of the torques is zero to find any unknown forces and lever arms.

LEARNING GOALS FOR SECTION B

When you have completed your study of Section B of this module, you should be able to do the following:

1. Given the mass and velocity of a body, calculate its momentum.

2. State the law of Conservation of Momentum.

3. Given the masses and velocities of two bodies before they experience a head-on inelastic collision, calculate the total momentum of the system, the velocity of the system after the collision, and the amount of kinetic energy lost during the collision.

LEARNING GOALS FOR SECTION C

When you have completed your study of Section C of this module, you should be able to do the following:

1. Given the average value of the force on a body or a system of bodies and the time interval during which the force acts, calculate the impulse of the force and the change of momentum of the system that experiences the force.

2. Define stress and yield stress.

3. Discuss at least two processes by which kinetic energy lost in an automobile collision can be converted to forms that do not result in damage to the cars involved.

4. Describe two techniques for minimizing the destructive effect of "second collisions" on the participants in an auto accident and know the physical principle responsible for their effectiveness.

SECTION A

Motion

WHY STUDY AUTO COLLISIONS?

The manufacture and sale of automobiles is big business. The health of the United States economy is closely coupled to the number of new automobiles purchased each year. Automobiles play an enormous role in the lives of Americans. We not only rely on cars for transportation to and from work and school, but almost all of our major forms of recreation require the kind of mobility provided by automobiles. At the same time that cars bring us convenience and pleasure, they introduce an element of tragedy into our national life. The toll in 1973 was 16 million accidents and 55,000 deaths. (This averages 150 deaths per day, so a three-day weekend toll of 500 is only a little above the average.)

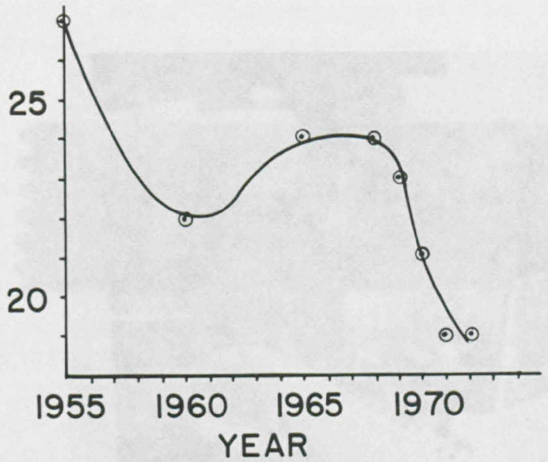

Figure 1. Motor vehicle deaths per million passenger miles.

Can anything be done to reduce the number of accidents, or to reduce the damage and injury caused by accidents? Already we have seen that reducing the speed limit on all U.S. highways has had a significant effect. Many researchers feel that much more can be done. We will discuss some research findings and ideas concerning the reduction of damage and injury in the last section of this module.

But, before you understand what happens in an accident and why some collisions cause more damage than others, you must learn something about the quantities that are used to describe motion and collisions.

What interesting questions might a study like this help to answer? First, we need to know which kinds of accidents are most common and which are most dangerous. What makes a collision dangerous? Is it high *velocity*, large *deceleration*, large *momentum*, great *energy*, large *interaction forces*, large *compressive stresses*, or short stopping time? Actually, all of these things are involved. They are related to each other through the *laws of dynamics*, and you need to know those laws if you want to understand automobile collisions.

However, don't think that knowing the laws of motion will enable you to design a crash-proof car or even to understand everything that happens during a collision. Real events are so complex that we cannot write equations that describe and predict everything that does or can happen. Simple theories provide complete understanding of simple events, but they may be the basis of a general understanding of more complex events. In particular, no theory is capable of predicting in detail the values of properties of matter, such as the strength of steel. So there is much need for experimental work to record carefully the details of complex events and measure accurately the properties of matter. Still, a simple theory is useful because it helps us to organize our knowledge and to predict the behavior of systems not yet built. Thus, it is of special importance to designers, including auto designers.

The principles of physics that are most relevant to automobile collisions are called *Conservation of Momentum* and the *Conservation of Energy*. These are, in turn, based on *Newton's laws of motion*. We will introduce each of these ideas by idealized situations and

experiments with simple systems. We will then apply these ideas to the study of real collisions. For example, if a Vega hits a Cadillac head-on, which car suffers the greater damage and which driver is more likely to be injured? The answers can be obtained by applying the Conservation of Momentum law, and results are generally understood in terms of this law. As another example, if all cars were about the same size, would we be safer with lighter cars or heavier cars? Dr. Patrick Miller, Head of the Structural Dynamics Section of the Transportation Safety Department of CALSPAN,* thinks we would be safer in small cars, and we will present his arguments later in the module. To begin your study with a bang, look at the film loop *Automobile Collisions.* This film, which shows several collisions, is part of a continuing program of research supported by the National Highway Traffic Safety Administration. A brief report of this work appears in the February, 1973, issue of *Scientific American* under the title "The Crashworthiness of Automobiles."

*The automobile safety research discussed in this module was conducted at the CALSPAN Corporation of Buffalo, New York, under Dr. Patrick M. Miller, with the support of the National Highway Traffic Safety Administration.

VELOCITY

To begin a serious study of automobile collisions, we need to develop a technical vocabulary and some mathematical relationships that will serve as general background. Section A is primarily concerned with providing this vocabulary and some simple relationships. We will begin by defining *velocity.* In this module, we will confine most of the discussion to motion along a straight line, and we will examine special kinds of simple motion that are important in technical work.

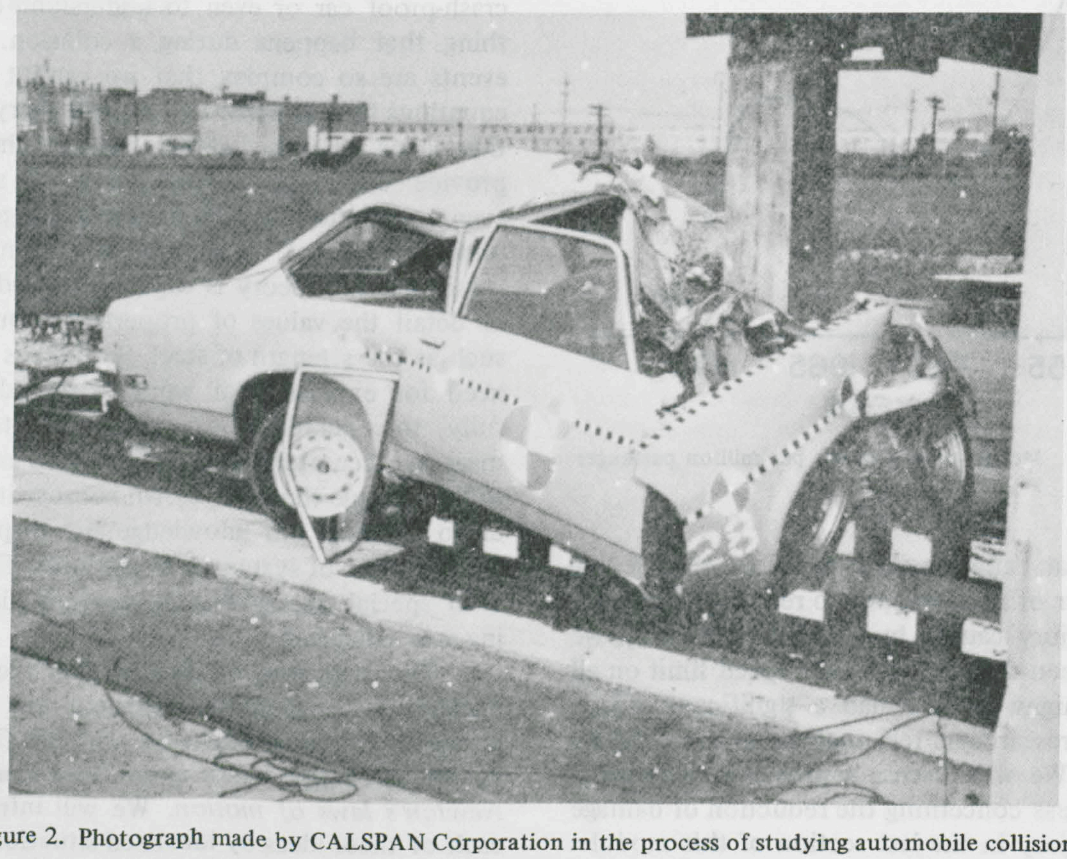

Figure 2. Photograph made by CALSPAN Corporation in the process of studying automobile collisions.

EXPERIMENT A-1. Motion—Uniform and Otherwise

I. Uniform Motion

You will have an opportunity to observe the motion of a metal glider along a level *air track*. In order to make precise statements about this motion, you need to measure the position of the glider at various known times. It is impossible to determine accurately the positions and times for the glider with a meter stick and a wrist watch. A better technique is to make a permanent record that you can examine at your leisure. One such method is to cause the glider to emit sparks in such a way that marks are made at known time intervals on a piece of paper tape along the track. Another good technique is to illuminate the glider with a flashing stroboscopic light and to record several successive positions of the glider on Polaroid film.

Use either technique to make a record of the motion of a glider over a level air track. The air track is used to study "ideal" motion because friction is minimized by the presence of an air film between glider and track. At this time we are interested in motion started by a push of some kind and continuing without outside influences. Your record should be similar to those shown in Figure 3.

II. Non-Uniform Motion

The motion you observed in Part I of this experiment is *uniform motion*. It is characterized by the fact that the cart moves an equal distance during each equal time interval. However, uniform motion is the exception rather than the rule. To produce motion that is not uniform, attach a piece of cardboard to your glider in such a way that air resistance is increased. Give the glider a push to get it started and make another record of its motion along the track. Figure 4A is a record of a similar non-uniform motion.

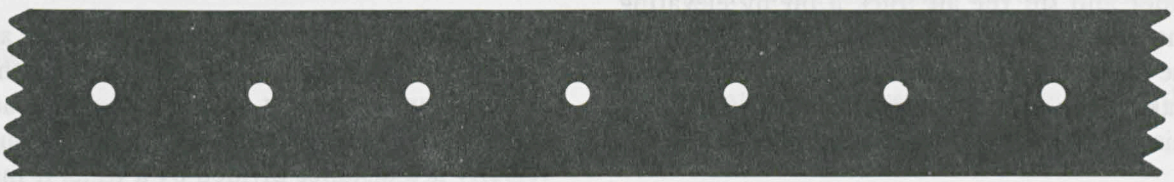

Figure 3A. A record of the motion of an air-track glider made by a spark-tape timer.

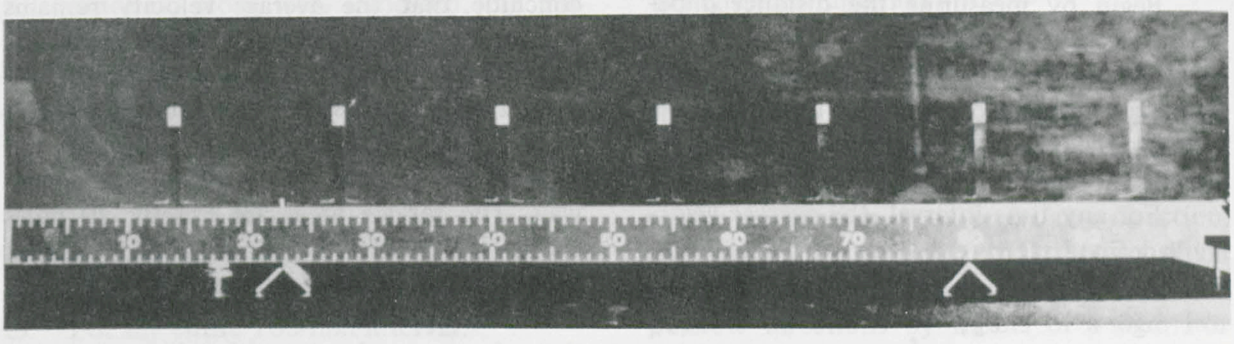

Figure 3B. A similar record, made by a stroboscopic light and a Polaroid camera.

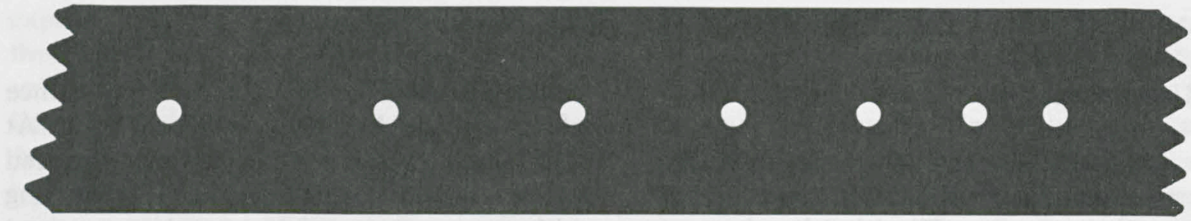

Figure 4A. A spark-timer record of slowing due to air resistance.

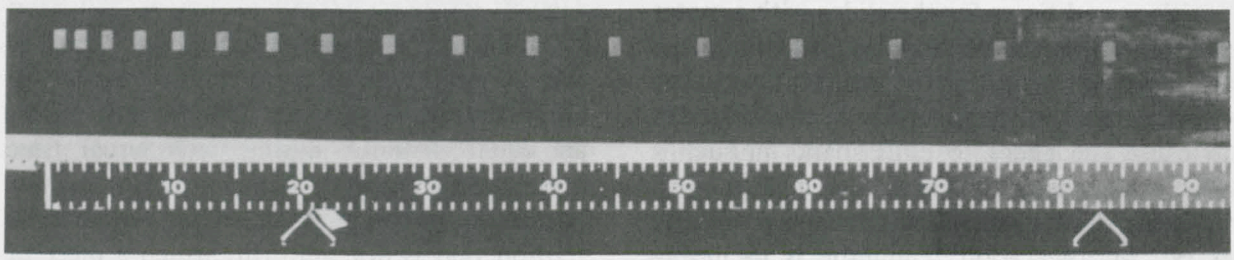

Figure 4B. A stroboscopic record of a glider uniformly accelerating down a tilted air track.

III. Uniformly Accelerated Motion

Now remove the cardboard from the glider and tilt the air track a bit by elevating one end a few millimeters. Put the glider on the track at the higher end and release it. Make a record of the motion. You might try this once more with a different tilt.

ANALYSIS OF EXPERIMENT A-1

I. Uniform Motion

Begin by measuring the distance d between successive positions of the glider for the level track. You will also need to know the time t between sparks or flashes of the stroboscope. Your teacher will help you to get this.

For any time interval, the *average velocity* is defined as:

$$V_{\text{av}} = \frac{d}{t} \qquad (1)$$

(Technically, what we have defined is called "speed" by physicists, and "velocity" has a direction associated with it. More about this later.)

The spark timer and the stroboscope are designed to produce equal time intervals; that is, the elapsed time t is the same for each interval between sparks or flashes. Since velocity is a distance divided by a time, it may be measured in meters per second (m/s), miles per hour (mi/h), or other distance units divided by time units. The distance d between successive sparks or flashes is also the same for each interval, at least within the accuracy of measurements made with a meter stick. We conclude that the average velocity remains constant as the glider moves along. We define uniform motion as motion with a constant velocity. Figure 3 shows this kind of motion.

Example 1. A man drives along an expressway. The service areas are exactly 20 miles apart. He passes the first service area at 9:20 A.M., the second area at 9:45 A.M., and arrives at the third at 10:10 A.M. He stops there for gas and a cup of coffee. He leaves again at 10:40 A.M. and passes the fourth service area at 11:00 A.M.

a. Based on the information given, what is the longest period of time during which the motion might have been uniform?

b. What is the average velocity during this period?

c. What was the average velocity during the time it took to move from the first to the fourth service area?

d. Could the driver ever have been arrested for breaking the 55 mi/h speed limit? How often?

Solution. First, use the definition to calculate the average velocity during each time interval.

For the first 25 minutes:

$$V_{\text{av}} = \frac{20 \text{ mi}}{(25/60) \text{ h}} = 48 \text{ mi/h}$$

For the next 25 minutes:

$$V_{\text{av}} = \frac{20 \text{ mi}}{(25/60) \text{ h}} = 48 \text{ mi/h}$$

For the next 30 minutes:

$$V_{\text{av}} = \frac{0 \text{ mi}}{(30/60) \text{ h}} = 0$$

For the next 20 minutes:

$$V_{\text{av}} = \frac{20 \text{ mi}}{(20/60) \text{ h}} = 60 \text{ mi/h}$$

a. Since no stops were indicated, the motion could have been uniform for the first 50 minutes.

b. For that 50-min interval:

$$V_{\text{av}} = \frac{40 \text{ mi}}{(50/60) \text{ h}} = 48 \text{ mi/h}$$

c. For the entire 100-min interval:

$$V_{\text{av}} = \frac{60 \text{ mi}}{(100/60) \text{ h}} = 36 \text{ mi/h}$$

d. Since the driver *averaged* 60 mi/h for the last 20 min, he surely exceeded 55 mi/h on the speedometer at least once during that time. If his motion was not uniform, he may have exceeded this velocity many other times during the trip. We don't have enough information to be sure.

Problem 1. During a trip across the Nebraska plains, a bored passenger in an automobile with a defective speedometer decided to count the telephone poles passed each minute in order to determine the speed of the car. The results for ten consecutive minutes were: 37, 37, 36, 35, 36, 36, 36, 36, 37, 39. Assume that the poles are exactly 40 meters apart.

a. What is the longest time during which perfectly uniform motion might have occurred?

b. What was the largest average velocity, in m/s, for a one-minute period?

c. What was the average velocity during the first two minutes?

d. What was the average velocity during the entire 10 minutes?

Problem 2. A car goes uphill at a steady 30 mi/h for 5 mi and then travels downhill at a steady 50 mi/h for another 5 mi. What is the average velocity over the time it took to travel this 10-mile stretch? **Careful:** use the definition given in Equation (1).

To specify velocity to a physicist's satisfaction, you must give both its numerical value (the magnitude of the velocity), and the direction of motion. This is important when the motion may be in any direction. Even when the motion occurs along a single straight line, we distinguish between the two possible directions by means of a sign. For example, we may choose to write 2 m/s toward the east as +2 m/s, and the same

speed toward the west as -2 m/s. A quantity, such as velocity, that has both a magnitude and a direction is called a *vector.*

In this module, we shall use both "velocity" and "speed" to mean speed, as the physicist defines it.

Acceleration

When the average velocity is not constant from one time interval to the next, we define a quantity that is a measure of the rate at which the velocity changes. The rate of change of velocity is called *acceleration.* Although the velocity may change in any manner, we shall examine only those motions in which the velocity changes by equal amounts during each time interval. Such motion is called *uniformly accelerated* and is seen in many cases in nature.

Uniformly Accelerated Motion

1. Using the tape (or photo) from Part III of the experiment, measure the distance traveled in each time interval with a meter stick. Record the distances in a table under the heading "d(m)."

2. Divide each value of d by the corresponding time interval t (in s). Record each quotient in a second column under the heading "V_{av}(m/s)."

3. Plot V_{av} versus time in the form of a *bar graph,* as shown in Figure 5. The bars take on the appearance of steps, and the heights of the steps (the difference between two successive values of V_{av}) are very nearly all the same.

If the "steps" in the bar graph were exactly equally high, one could calculate the *rate* at which the velocity changes by dividing the height of a step (which is the velocity change) by the width of the step (the corresponding time interval). The rate at which the velocity changes during a time interval *is* the *average acceleration* during that interval. Mathematically, we write

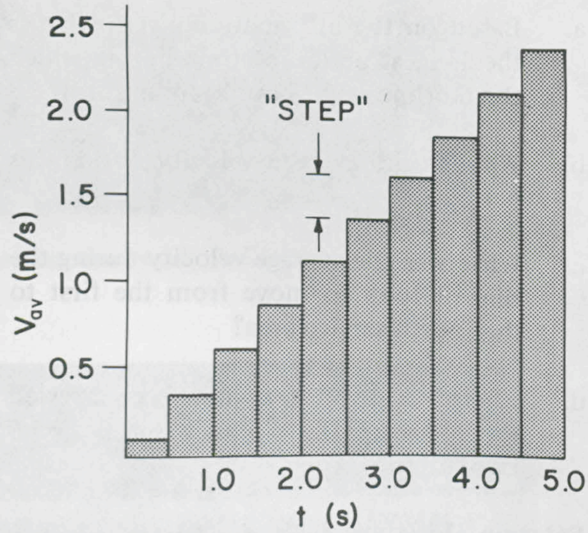

Figure 5.

$$a_{av} = \frac{\text{change in velocity during time interval}}{\text{time interval}}$$

For the bar graph (Figure 5) this is the same as saying

$$a_{av} = \frac{\text{step height}}{\text{step width}} \tag{2}$$

A better procedure, which works even if the step heights are not quite the same, is to draw a straight line which passes as closely as possible through the midpoint of the top of each step. When you do this, you are changing from a graph of V_{av} *during a time interval* versus time to a graph of velocity V *at a particular time* versus the corresponding time. You are, in effect, saying that the average velocity during a time interval is the same as the actual velocity at the midpoint of the time interval. This will always be true if the acceleration does not change during the time interval.

4. Make a V versus t graph by drawing a straight line through the midpoints of the steps in your V_{av} versus t graph as in Figure 6.

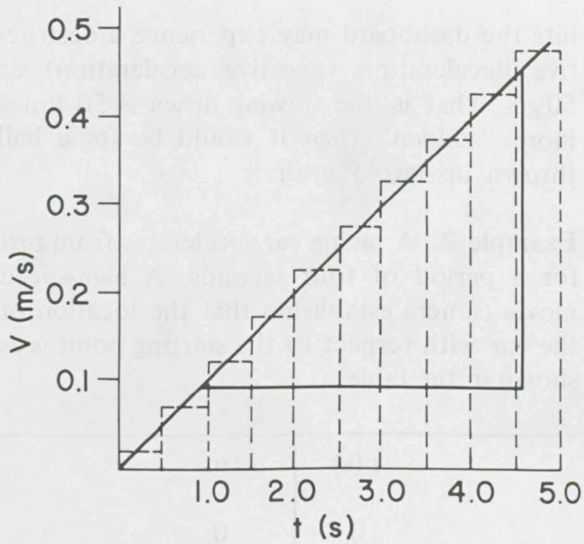

Figure 6.

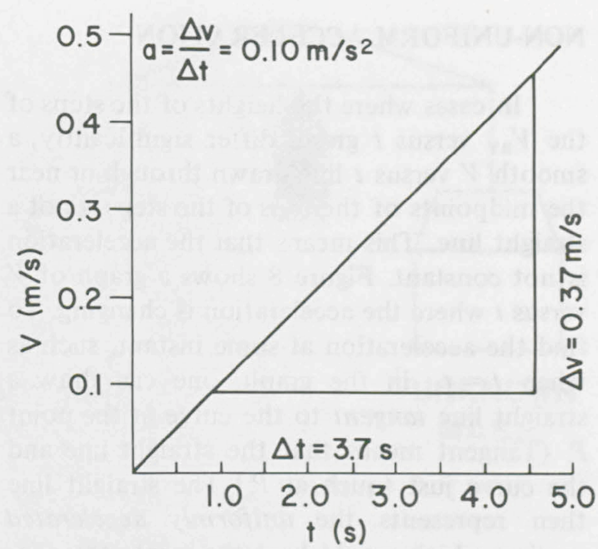

Figure 7.

5. As shown in Figure 6, draw a right triangle by drawing a vertical line and a horizontal line from *any* two points on the V versus t line. Figure 7 is a somewhat simplified version of Figure 6, showing the values of the legs of the triangle.

The vertical side of the triangle is a change in velocity, ΔV, while the horizontal side is the corresponding change in time, Δt. (The Greek letter *delta* [Δ] is used to indicate the *change* in a quantity.) Then the acceleration is given by the *slope* of the V versus t graph. That is:

$$a_{\text{av}} = \frac{\Delta V}{\Delta t} = \text{slope} \qquad (3)$$

The units for acceleration are meters per second per second (m/s^2)*. The kind of motion represented by Figure 6, a graph where all the steps are equally high, is called *uniformly accelerated motion.*

Problem 3. Using the constructions you have just made, calculate the acceleration of your glider.

*These are metric units which are part of an internationally agreed upon system of units. The entire system is called *SI (Système International)* and is gaining wide acceptance throughout the world.

NON-UNIFORM ACCELERATION

In cases where the heights of the steps of the V_{av} versus t graph differ significantly, a smooth V versus t line drawn through or near the midpoints of the tops of the steps is not a straight line. This means that the acceleration is not constant. Figure 8 shows a graph of V versus t where the acceleration is changing. To find the acceleration at some instant, such as when $t = t_1$ in the graph, one can draw a straight line *tangent* to the curve at the point P. (Tangent means that the straight line and the curve just touch at P.) The straight line then represents the *uniformly accelerated* motion which would have the same values for both velocity and acceleration at the point P as does the curve.

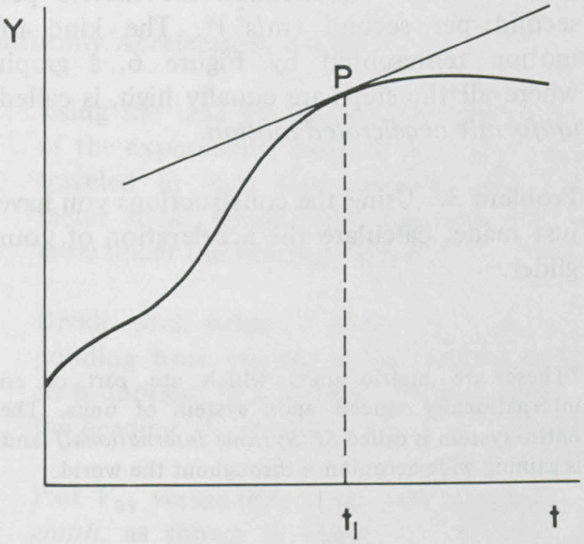

Figure 8.

GRAVITATIONAL ACCELERATION

Ignoring the drag of air resistance, objects that fall freely near the surface of the earth are observed to accelerate downward at a constant $9.8\ \mathrm{m/s^2}$. Even an object that is moving upward under the influence of gravity alone experiences this acceleration; that is, its upward velocity decreases at the rate of 9.8 m/s every second. The acceleration produced by gravity is called *g*. Because it is so well known and widely experienced, it is common to measure other accelerations in terms of *g*'s. In a collision, a passenger who hits the dashboard may experience a destructive deceleration (negative acceleration) of 50 *g*'s. That is, the slowing down is 50 times more "sudden" than it would be for a ball thrown up into the air.

Example 2. A racing car accelerates from rest for a period of four seconds. A high-speed movie camera establishes that the location of the car with respect to the starting point is as shown in the table.

t (s)	*d* (m)
0	0
1	5
2	20
3	45
4	80

a. Was the acceleration uniform?
b. What is the value of the average acceleration during this 4-s period?
c. How fast is the car moving after 4 s?

Solution.

a. The average velocity during each successive second of motion is found by dividing the distance moved by the time. The results are 5 m/s, 15 m/s, 25 m/s, 35 m/s. Since the difference between any two successive values of V_{av} is the same, 10 m/s, this acceleration is uniform.

b. The acceleration is calculated from Equation (3) as follows:

$$a = \frac{\Delta V}{\Delta t}$$

$$= \frac{10\ \mathrm{m/s}}{1\ \mathrm{s}} = 10\ \mathrm{m/s^2}$$

Note that, for this straight-line graph (constant acceleration), any other time interval gives the same answer:

$$a = \frac{20\ \mathrm{m/s}}{2\ \mathrm{s}} = \frac{30\ \mathrm{m/s}}{3\ \mathrm{s}} = \frac{40\ \mathrm{m/s}}{4\ \mathrm{s}}$$

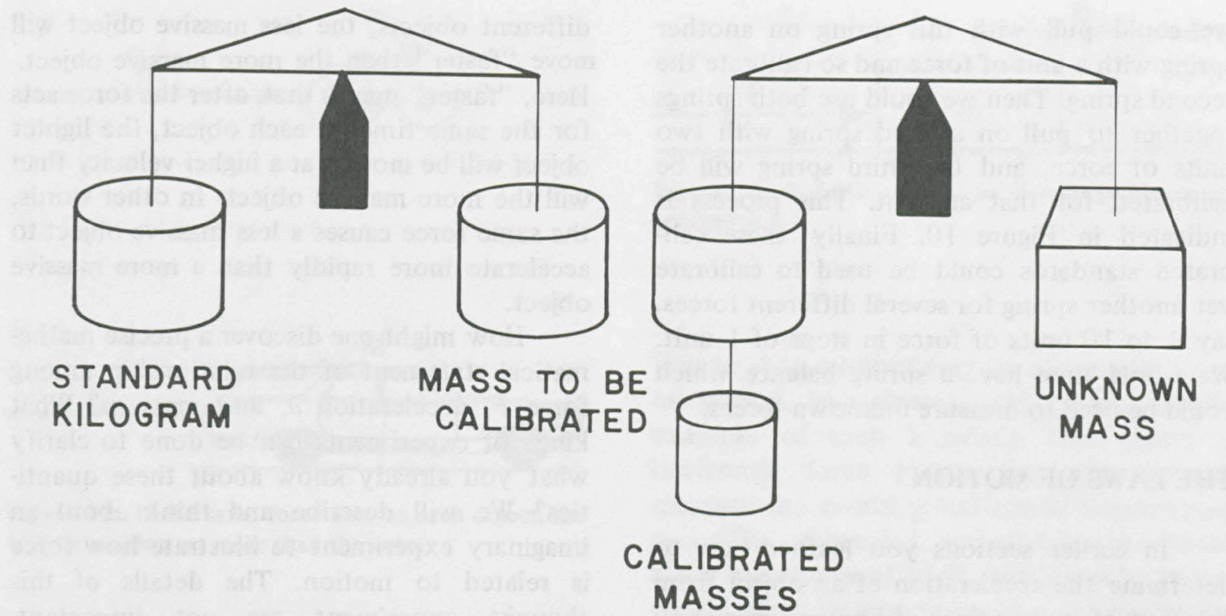

Figure 9. Unknown masses are measured by comparing them to calibrated sets of masses. The calibrated masses were at some time compared, probably indirectly, to the standard mass.

c. By rearranging Equation (3), we see that, for constant acceleration, the change in velocity ΔV is given by

$$\Delta V = a\Delta t$$

In this case, $a = 10\ \text{m/s}^2$, $\Delta t = 4$ s, and $\Delta V = 10\ \text{m/s}^2 \cdot 4\ \text{s} = 40$ m/s. Since the car started from rest, the change in speed ΔV during four seconds is equal to the speed at the end of the four seconds.

MASS

Mass is a measure of the quantity of matter in an object. Although you may intuitively understand what mass is, it is a difficult thing to define. Physicists use an *operational definition* of mass. An operational definition is a prescription for a procedure which, when carried out, results in a value for the quantity that is being defined. In this case, the procedure is to use an *equal-arm balance* to produce a balance between the unknown mass and a calibrated set of masses. The essential part of the procedure–but one that you need not worry about–is the calibration of the set of masses. That is accomplished with an equal-arm balance, as indicated in Figure 9, provided that there exist standard masses. What is a standard mass? By definition, one kilogram (1 kg) is the mass of a certain piece of platinum-iridium alloy which has been accepted as the standard kilogram by all countries of the world. It is kept in a vault near Paris, France. Other standard kilograms that have been carefully compared with *the* standard kilogram are available in each country, and sets of calibrated masses that have been compared with these standards are still more widely available. Any unknown mass can be balanced by some combination of calibrated masses. Using this procedure, a value for mass can be assigned to every object.

FORCE

Every push or pull is a *force.* This statement is adequate to give you a good idea of the meaning of force. However, for precise work we need a more precise definition. Again, we will use an operational definition. The measuring instrument will be an ordinary coil spring. We could arbitrarily select a certain extension (amount of stretch) of the spring and define a unit of force as the force required to hold the spring in this extended position. By marking this position, we could use the spring to measure a unit of force. Now

we could pull with this spring on another spring with a unit of force and so calibrate the second spring. Then we could use both springs together to pull on a third spring with two units of force, and the third spring will be calibrated for that amount. This process is indicated in Figure 10. Finally, these calibrated standards could be used to calibrate yet another spring for several different forces, say 1 to 10 units of force in steps of 1 unit. We would then have a spring balance which could be used to measure unknown forces.

THE LAWS OF MOTION

In earlier sections you learned how to determine the acceleration of an object from a record of its positions at known times. You also discovered that there are simple instruments which can be calibrated to measure the masses of objects and the values of forces exerted on them. You know from daily experience that if the same force is applied to objects of different mass, different effects are produced. If you apply the same force to two different objects, the less massive object will move "faster" than the more massive object. Here, "faster" means that, after the force acts for the same time on each object, the lighter object will be moving at a higher velocity than will the more massive object. In other words, the same force causes a less massive object to accelerate more rapidly than a more massive object.

How might one discover a precise mathematical statement of the relationship among force F, acceleration a, and mass m? What kinds of experiments can be done to clarify what you already know about these quantities? We will describe and think about an imaginary experiment to illustrate how force is related to motion. The details of this *thought experiment* are not important. Similar real experiments can be designed and have been done many times. When such experiments are carefully performed, the results are always the same. Since no careful experiment has ever produced an exception to these results, we gain confidence in them and treat them as a "law of nature."

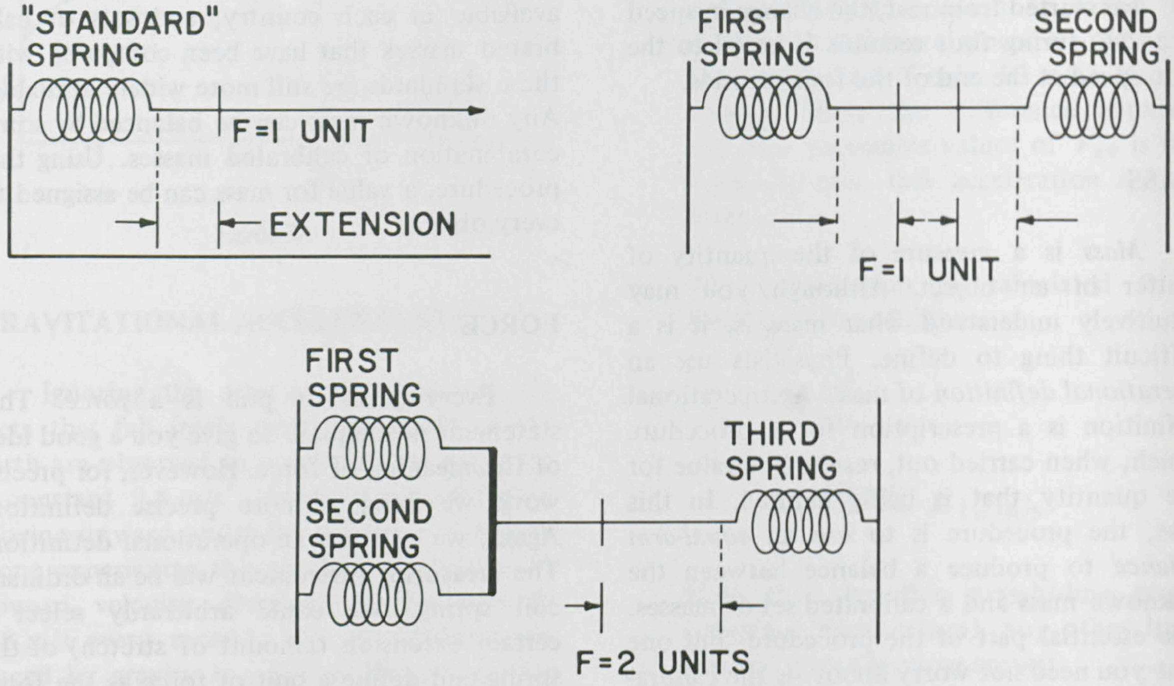

Figure 10. The "standard" spring is used to calibrate other springs.

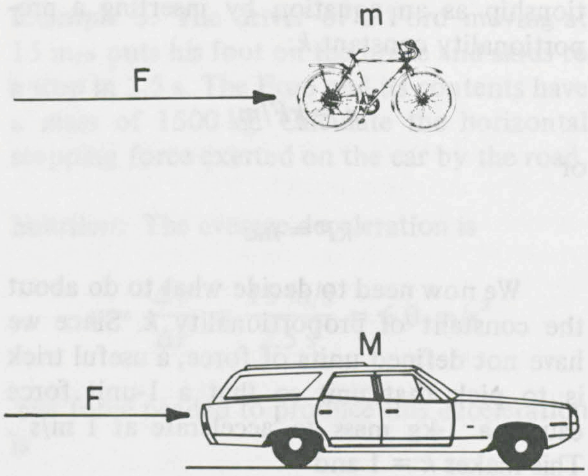

Figure 11. If the same force acts on each object, the bicycle accelerates more than the auto.

In our thought experiment, we shall think of the motions of objects when forces are applied to them. If we can measure the force, the mass, and the acceleration for several cases, we should be able to find a relationship among the three quantities (*variables*). However, when one seeks a relationship among three or more variables, it is essential that the experimental design allows the variables to be *controlled*. This means that one quantity must be held constant while the second is changed in a controlled way; then the corresponding changes in the third quantity are measured.

In our imaginary experiment, we will first hold the mass constant and see how changing the force affects the resulting acceleration. Then we will hold the force constant and see how changing the mass affects the acceleration. We could then hold acceleration constant and see how changing the mass affects the force necessary to produce that constant acceleration. However, this last step is not necessary and would merely serve to confirm the relationship implied by the first two sets of measurements. For this reason, the first two sets are all we will discuss in our thought experiment, and are all that are necessary in a real experiment.

Begin by imagining a mass m which is free to slide without friction along a horizontal surface. (An air-track glider is one simple example of such a mass.) Then apply a horizontal force F_1 to the mass m and measure the resulting horizontal acceleration a_1. (The calibrated spring balance would allow measurement of F_1 and a strobe photo would allow measurement of a_1.) Then apply a larger force F_2 to the same mass and measure the resulting acceleration a_2. Proceeding in this way, data may be collected. The results of real experiments show that the data would produce a graph like that of Figure 13. The graph of applied force F versus acceleration a of the constant mass m is found to be a straight line passing through the origin. This means that the acceleration is proportional to the applied force. That is, the greater the applied force, the greater the resulting acceleration. Mathematically, this can be written as:

$$a \propto F \text{ (for constant } m)$$

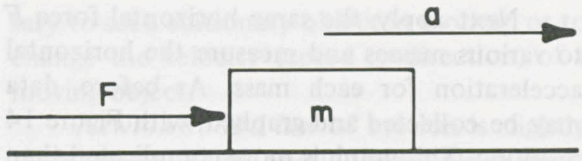

Figure 12. A force applied to a mass produces an acceleration.

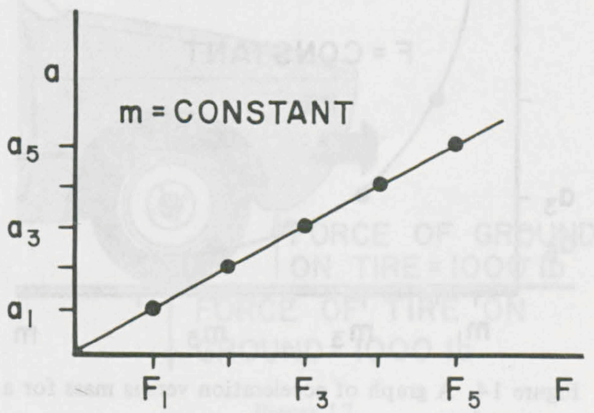

Figure 13. A graph of acceleration versus force for a constant mass.

Next apply the same horizontal force F to various masses and measure the horizontal acceleration for each mass. As before, data may be collected and graphed, with Figure 14 resulting. This graph is more complicated than one might like, and it is not as easy to interpret as was Figure 13. When this happens, it is often fruitful to try to produce a straight-line graph by plotting a versus some other quantities such as m^2 or $\sqrt{m}$. For a graph like Figure 14, $1/m$ or $1/m^2$ are likely candidates. In fact, the graph of a versus $1/m$ *does* produce a straight line, as shown in Figure 15. This means that acceleration is proportional to the inverse of the mass, or:

$$a \propto 1/m \quad \text{(For constant } F\text{)}$$

Thus we have determined that the acceleration is proportional to the applied force if the mass is constant, and proportional to the inverse of the mass if the force is constant. It is reasonable to expect that the acceleration actually is proportional to the product of applied force and inverse mass. That is,

$$a \propto F \cdot \frac{1}{m}$$

We can then write the proportionality relationship as an equation by inserting a proportionality constant k:

$$a = kF/m$$

or

$$kF = ma$$

We now need to decide what to do about the constant of proportionality k. Since we have not defined units of force, a useful trick is to pick that unit so that a 1-unit force causes a 1-kg mass to accelerate at 1 m/s^2. This makes $k = 1$ and

$$F = ma \qquad (4)$$

This equation is *Newton's second law* of motion, and is one of the most basic in physics. The resulting unit of force, kg·m/s^2, is so frequently encountered that it is given the name *newton* (N):

$$1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$$

The general validity of Equation (4) for all values of F and m is an experimental result which has been verified by performing many experiments much like the one described above. It is called a "law" only because every experiment has verified it.

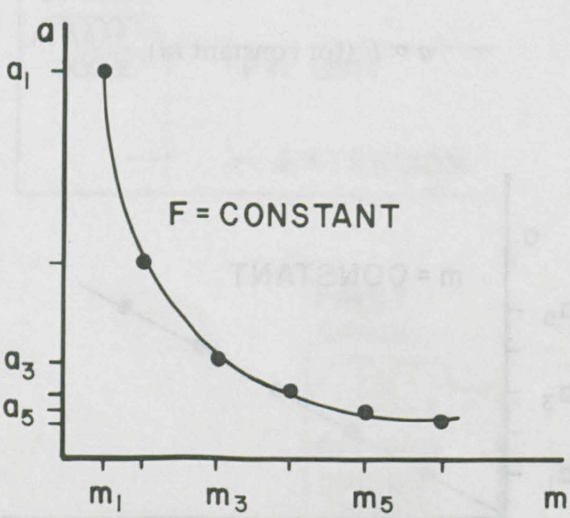

Figure 14. A graph of acceleration versus mass for a constant force.

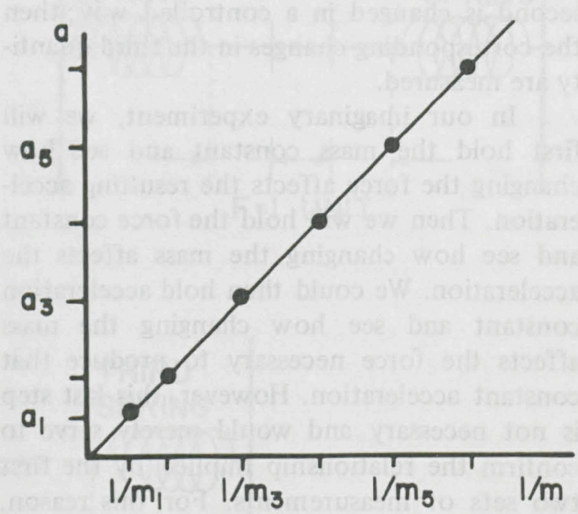

Figure 15. Graph of acceleration versus the inverse of mass at constant force.

Example 3. The driver of a Ford moving at 15 m/s puts his foot on the brake and skids to a stop in 2.5 s. The Ford and its contents have a mass of 1500 kg. Calculate the horizontal stopping force exerted on the car by the road.

Solution. The average deceleration is

$$a = \frac{\Delta V}{\Delta t} = \frac{15 \text{ m/s}}{2.5 \text{ s}} = 6.0 \text{ m/s}^2$$

The force needed to produce this deceleration is

$$F = ma$$

$$= 1500 \text{ kg} \times 6.0 \text{ m/s}^2 = 9000 \text{ kg} \cdot \text{m/s}^2$$

$$= 9000 \text{ N}$$

Problem 4. Suppose that the Ford mentioned in Example 3 hits a wall while moving at 15 m/s. The driver's seat belt holds, and he is brought safely to rest as the front end of the car crumples and shortens by 0.9 m. The mass of the driver is 80 kg.

a. Assuming that the deceleration of the rest of the car is uniform as the front end of the car crumples, what is the average velocity of the driver as he comes to rest?

b. How long does it take him to come to rest? (Recall that $d = V_{av}t$)

c. What is the deceleration of the driver during the crash?

d. What force does the seat belt exert on him?

Newton's first law of motion is sometimes (loosely) thought of as a special case of the second law. If the *net* force F on an object is zero, the object experiences *no* acceleration. In other words, a force is necessary to set a stationary object in motion, or to change the velocity (speed *or* direction) of a moving object.

Newton's third law of motion is slightly more subtle. Think about the simple example of the spring balance used in our definition of force. Suppose you pull the spring to the 2-N mark as shown in Figure 16. To do this, you must exert a force of 2 N on the spring. Newton's third law says that the spring exerts a (reaction) force of 2 N on your hand. That is, for every applied force, called an *action* force, there is an equal and oppositely directed *reaction* force. Forces do not occur alone; they always occur in action-reaction pairs. A force must be exerted *on* something. In turn, the object upon which the force is exerted exerts an equal and opposite reaction force. Figure 17 shows another example. An automobile tire exerts a force on the ground and the ground exerts an equal and oppositely directed force on the tire.

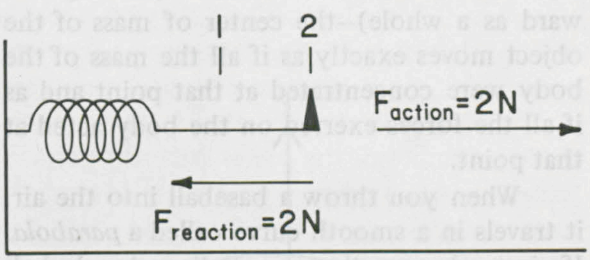

Figure 16.

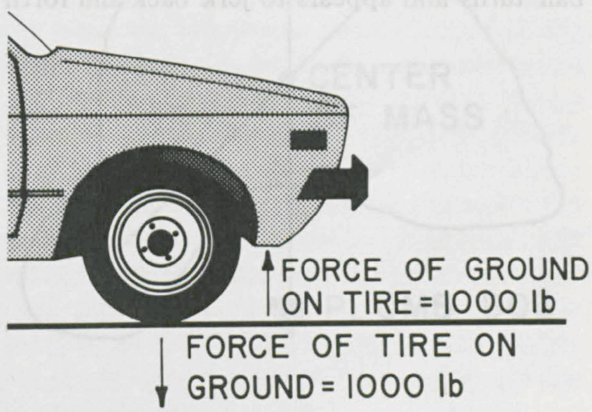

Figure 17.

CENTER OF MASS

We often speak of "head-on" collisions. In such collisions, either the wreckage comes to rest fairly quickly, or the cars move forward or backward along the original line of motion. However, if two cars collide "off center," the wreckage spins before it comes to rest. What is the condition that distinguishes between "head-on" and "off center"?

A car that strikes a telephone pole a glancing blow or that runs off the road into a ditch may roll over. The likelihood that this will happen is reduced by driving a car that is "close to the road" or by loading the undercarriage of the car with extra mass. Why is this? Are "off-center" collisions and "roll-overs" related in any way?

It turns out that there is a point associated with every body which has special properties. It is called the *center of mass.* When an object moves under the influence of forces–even in complicated ways that may include simultaneous *rotational motion* (turning) and *translational motion* (moving forward as a whole)–the center of mass of the object moves exactly as if all the mass of the body were concentrated at that point and as if all the forces exerted on the body acted at that point.

When you throw a baseball into the air, it travels in a smooth curve called a *parabola.* If, instead, you throw a hollow beach ball with a piece of lead taped to its surface, the ball turns and appears to jerk back and forth as it moves forward. Close observation reveals that there is a point between the center of the beach ball and the piece of lead which moves along the same parabolic path traced by the moving baseball, provided both balls start with the same velocity (and if air resistance is small). This special point on the loaded beach ball is its center of mass. The baseball does not wobble when it spins because its center of mass is located at its geometric center. Figure 18 illustrates the same idea for the (perhaps) more familiar case of a baseball and a baseball bat.

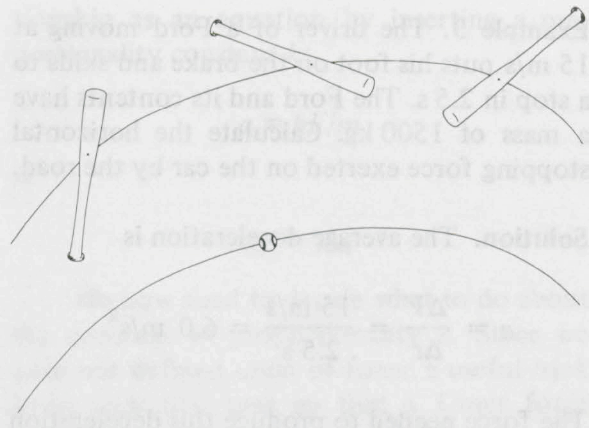

Figure 18.

There are several ways to locate the center of mass for a particular object. The following experiments will illustrate some of these. Why these methods work, as well as some more precise methods, will be discussed in later sections of this module.

EXPERIMENT A-2. Locating Centers of Mass

Method 1

Here is a simple way to locate the center of mass of a flat object, such as a large, irregularly shaped piece of heavy cardboard of plywood. Drill holes at several different points near the edge. Suspend the board by means of a string tied through one of the holes so that it hangs freely. When the object has stopped moving, its center of mass will lie on a vertical line through the point of support. To locate such a vertical line, hang a *plumb bob* from the support so that the string supporting the bob passes across the face of the board. Draw a line on the face of the board along this string. (See Figure 19.) This will give you a line on which the center of mass must be. Repeat this procedure using a different hole in the board. Since both lines contain the center of mass, the point where the two lines intersect *is* the center of mass. You may try other holes to convince yourself that all "plumb" lines pass through the center of mass.

Method 2

Tape an object, with a mass of 100 or 200 g, securely to one end of a meter stick. Tape colored pieces of paper on one side of the meter stick every 5 cm, starting from the 50-cm mark and going to the end where the extra mass is taped; each piece should be a different color. Take the meter stick outdoors and throw it gently into the air, giving it an end-over-end motion but keeping the side with the colored pieces of paper facing in one direction. Persons watching the motion of the colored paper will see each piece travel in a small circle caused by the rotating motion, superimposed on a large arc that traces the overall motion of the stick. The piece of paper that is least influenced by the rotating motion, and therefore travels in the smoothest large arc, is closest to the center of mass. Saying this a little differently, the whole thing rotates about the center of mass as if there were an axis through that point.

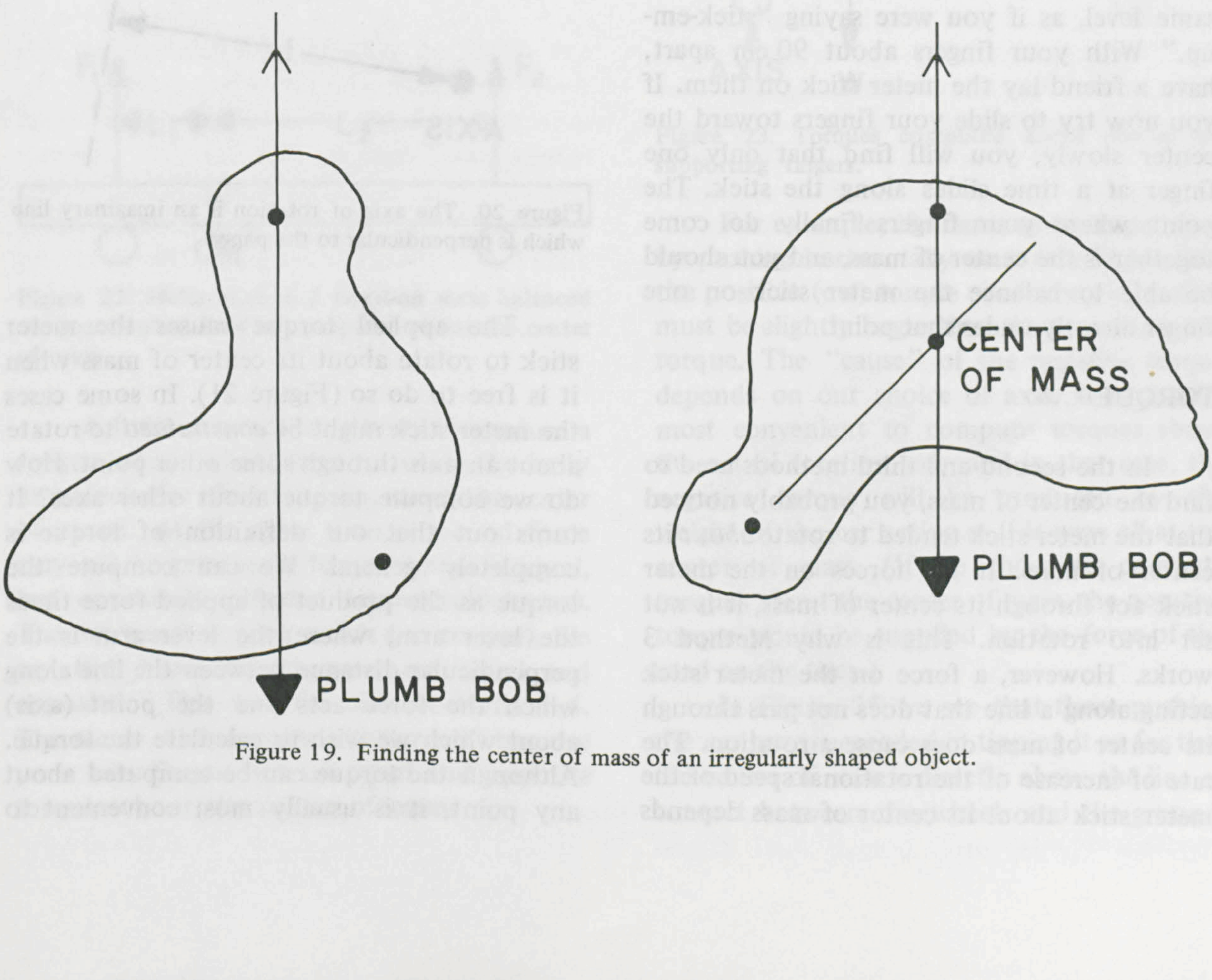

Figure 19. Finding the center of mass of an irregularly shaped object.

Method 3

Use the same meter stick with a mass taped on one end as in Method 2. Lay it on a flat, smooth table in a horizontal position. Use a hammer or a ruler to strike the meter stick with sharp blows at various points along its length in a direction perpendicular to its length. In general, the stick undergoes motion as a whole in the direction of the blow and it rotates as well. A blow struck away from you as you face the object and near the left end causes a clockwise rotation. A blow directed away from you that hits the stick near its right end causes a counterclockwise rotation. At some point along the stick, a blow causes no rotation. This point is the center of mass. (This suggests that an automobile will not rotate after it is struck by another if the motion of the second is directed straight at the center of mass of the first.)

Method 4

Use the same meter stick with attached mass used in Methods 2 and 3. Hold out your two index fingers horizontally and at the same level, as if you were saying "stick-em-up." With your fingers about 90 cm apart, have a friend lay the meter stick on them. If you now try to slide your fingers toward the center slowly, you will find that only one finger at a time slides along the stick. The point where your fingers finally do come together is the center of mass, and you should be able to balance the meter stick on one finger directly under that point.

TORQUE

In the second and third methods used to find the center of mass, you probably noticed that the meter stick tended to rotate *about* its center of mass. If the forces on the meter stick act through its center of mass, it is not set into rotation. This is why Method 3 works. However, a force on the meter stick acting along a line that does not pass through its center of mass does cause a rotation. The rate of increase of the rotational speed of the meter stick about its center of mass depends not only on the applied force, but also on the distance between the center of mass and the line along which the force acts. The perpendicular distance between the line along which the force is directed and the point about which the torque is calculated is called the *lever arm* of the force. A force applied along a line not through the center of mass exerts a *torque* about the center of mass. The magnitude of the torque is defined as the product of the force times the lever arm. That is,

$$T = FL \quad (5)$$

where L is the perpendicular distance between the center of mass and the line along which the force acts. (See Figure 20.)

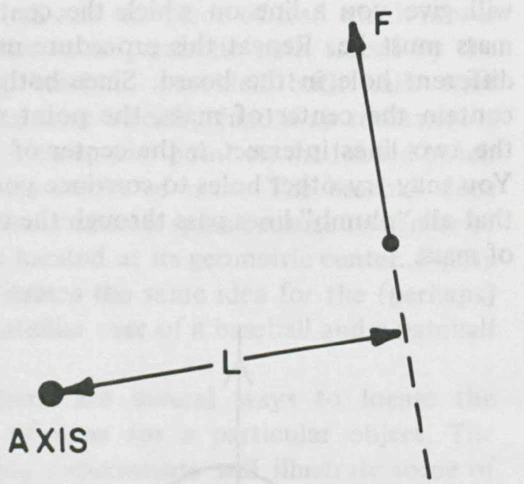

Figure 20. The axis of rotation is an imaginary line which is perpendicular to the paper.

The applied torque causes the meter stick to rotate about its center of mass when it is free to do so (Figure 21). In some cases the meter stick might be *constrained* to rotate about an axis through some other point. How do we compute torque about other axes? It turns out that our definition of torque is completely general. We can compute the torque as the product of applied force times the lever arm, where the lever arm is the perpendicular distance between the line along which the force acts and the point (axis) about which we wish to calculate the torque. Although the torque can be computed about any point, it is usually most convenient to

compute the torque about the axis of rotation.

If more than one force acts on a body, the torques produced by each force add together to produce a single net torque on the body. In general, torques are vectors, and they must be added using the rules for vector addition. However, if the forces acting lie in a single plane (for example, the surface of a lab table or the surface of a road), then the addition of torques is done by the rules of algebra. We assume that this is always the case throughout this module.

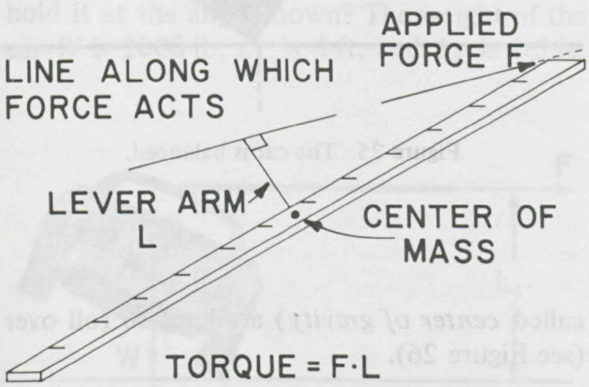

Figure 21. Finding torque about the center of mass.

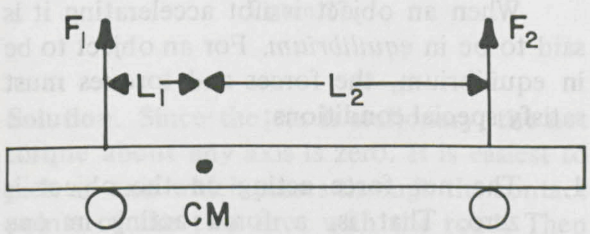

Figure 22. Meter stick and taped-on mass balanced on two fingers. Torques computed about the center of mass.

A force may act on a body to produce a torque which would tend to rotate the body in a clockwise direction (say, about the center of mass). At the same time, a second force may act to produce a torque which is equal, but oppositely directed (counterclockwise). The net result is no rotation (no torque). We say that these two torques are equal and opposite. This was the case in Method 4. The meter stick remained "balanced" because your two fingers set up equal and opposite torques about the center of mass.

For any object that is not accelerating, the sum of all torques about *any* point is equal to zero. For example, you might choose to compute torques about the finger on the left in Figure 23. In this case, the equal and opposite torques are produced by the other finger and by the force of gravity (weight) which acts downward at the center of mass (Figure 23). No torque is produced by F_1 since it acts through the point about which torques are computed (its lever arm is zero).

In actual calculations, you must pay careful attention to the directions of torques by keeping track of their signs. In this module, we choose to say that a *positive* torque is one which would produce a *clockwise* rotation and a *negative* torque produces a *counterclockwise* rotation. (The opposite convention is sometimes used.)

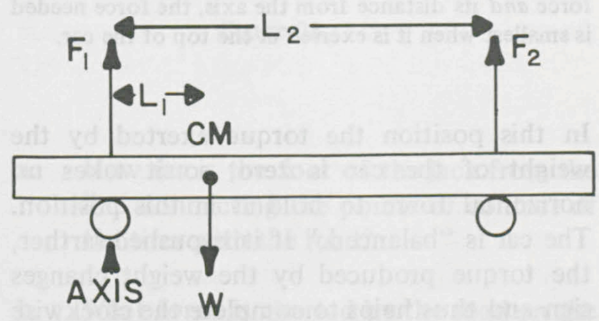

Figure 23. Torques computed about one of the supporting fingers.

For example, if a car is to be tipped over by pushing horizontally on it (see Figure 24), the positive (cw) torque produced by pushing must be slightly larger than the negative (ccw) torque. The "cause" of the negative torque depends on our choice of axis. It is usually most convenient to compute torques about the actual rotation axis, and in that case, the negative torque will be produced by the weight of the car acting as if it were all at the center of mass. (If we chose to compute torques about the center of mass, the negative torque would be supplied by the force of the road on the tires.)

In Figure 25 we see that those pushing the car have succeeded in tipping it so far that the center of mass is directly above the line of contact between the wheels and the ground.

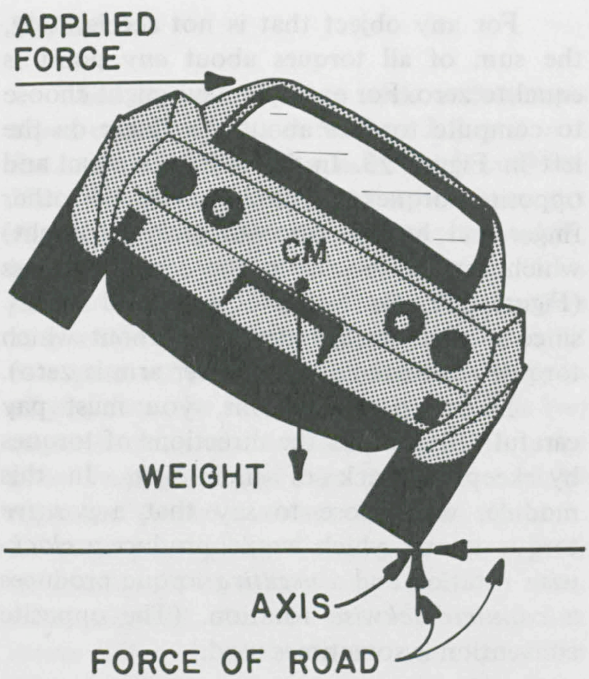

Figure 24. Since the tipping torque depends on the force *and* its distance from the axis, the force needed is smallest when it is exerted at the top of the car.

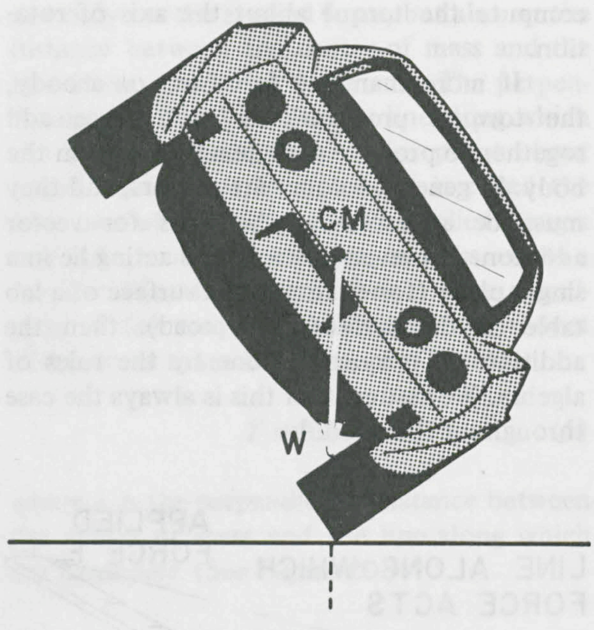

Figure 25. The car is balanced.

In this position the torque exerted by the weight of the car is zero, so it takes no horizontal force to hold it in this position. The car is "balanced." If it is pushed farther, the torque produced by the weight changes sign and thus helps to complete the clockwise turn. The angle through which the car must be turned before reaching the position shown in Figure 25 is small if the center of mass is high above the road. This should make it clear why cars that have a low center of mass (also called *center of gravity*) are hard to roll over (see Figure 26).

EQUILIBRIUM

When an object is not accelerating it is said to be in *equilibrium.* For an object to be in equilibrium, the forces and torques must satisfy special conditions:

1. The net force acting on the object is zero. That is, a force acting in one

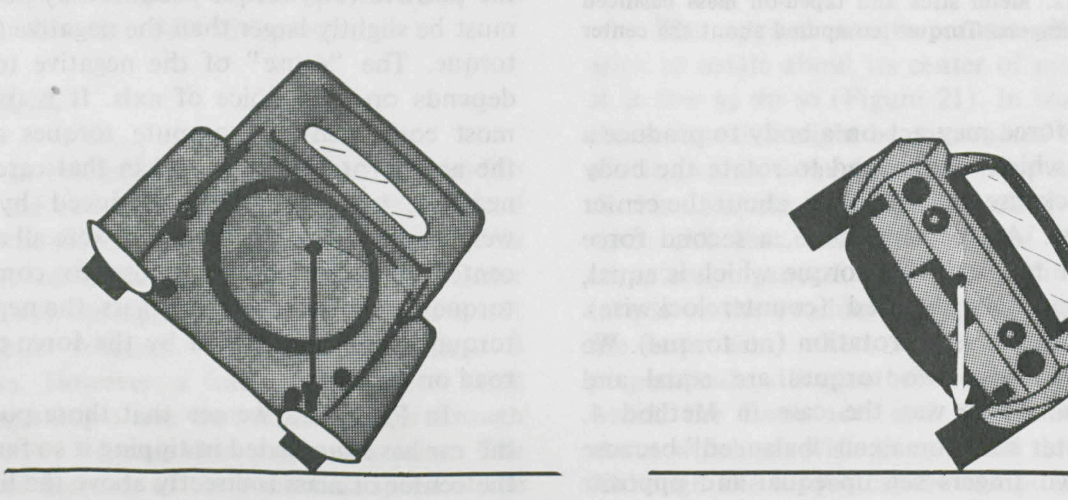

Figure 26. The old car, with a high center of gravity, balances at a smaller angle and thus tips over more easily than the modern car.

direction is balanced by a force acting in the opposite direction.

2. The net torque acting on the object is zero. That is, a torque acting in one direction is balanced by a torque acting in the opposite direction. This is true no matter what point is used for calculation of the torques.

Example 4. A car is tilted over on two wheels by attaching a rope to the center of its roof and pulling horizontally, as shown in Figure 27. How much applied force F is required to hold it at the angle shown? The weight of the car W is 2000 lb, L_1 is 4 ft, and L_2 is 1.5 ft.

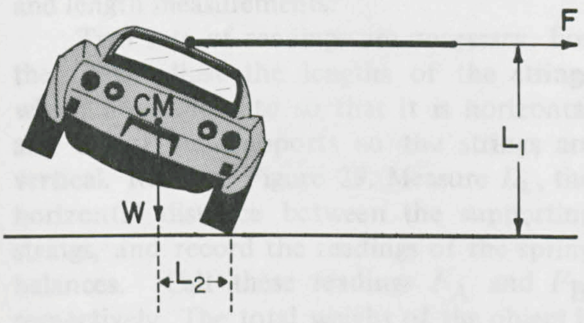

Figure 27.

Solution. Since the car is stationary, the net torque about any axis is zero. It is easiest to pick an axis which passes through the contact points of the two tires with the road. Then the torque exerted by the applied force is $+FL_1$ and by the weight it is $-WL_2$. Thus

$$FL_1 - WL_2 = 0$$

Solving this for the force

$$F = \frac{WL_2}{L_1} = \frac{2000 \text{ lb} \times 1.5 \text{ ft}}{4 \text{ ft}} = 750 \text{ lb}$$

One interesting point is that, since the car is not accelerating as a whole, the net force in any direction is also zero. This means that there is a force of 750 lb to the left and a force of 2000 lb upward, both exerted by the road on the tires.

Problem 5. The car of the preceding example is tilted to the same angle as before, this time by jacking up one side, as shown in Figure 28. The force exerted by the jack is 1000 lb.

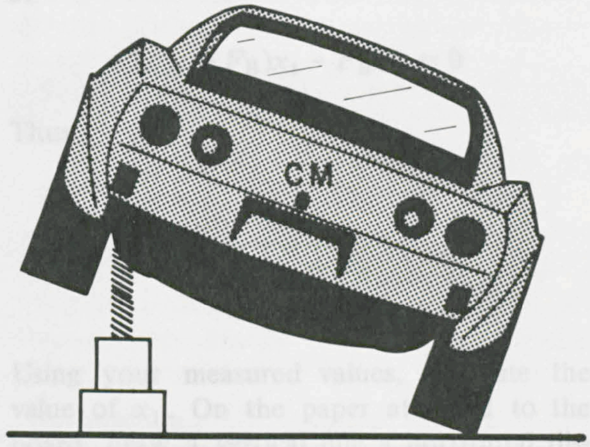

Figure 28.

a. How far is the foot of the jack from the line connecting the points of contact of the tires with the road?

b. What force is exerted by the road on the tires?

The discussion of torques should help you to understand why you were able to find the center of mass of the "weighted" meter stick by supporting it on two fingers and slowly moving the fingers toward each other (Experiment A-2, Part 4). The two fingers supporting the stick support unequal portions of the total weight of the stick. The *frictional* force between a finger and the stick (the force which "resists" sliding) depends on the weight supported by the finger. The greater the weight, the greater will be the frictional force. The finger farther from the center of mass always supports a smaller share of the total weight, and thus experiences less frictional force than the finger closer to the center of mass. The result is that the finger farther from the center of mass moves toward the center of mass until *it* supports the larger portion of the weight (and thus is closer to the center of

mass). Then the other finger slides more easily and the process is repeated.

These same equilibrium methods may be used to find the center of mass of an automobile.* The next experiment will show you how this may be done.

*More theory on locating the center of mass may be found in the Appendix at the back of the module.

EXPERIMENT A-3. Locating the Center of Mass of an "Automobile"

In this experiment you will use a large board shaped like an automobile and having holes from which the board may be suspended. A mass may be attached in the hood area to simulate the weight of a motor. (It may also be attached at the back to simulate a rear-engine auto.)

Attach a large piece of paper to the board so that it covers most of the area near the center of the board. Tie strings from the hooks to the movable parts of spring balances which are firmly supported from directly above. Attach the motor weight at the hood or the rear compartment. The object of the experiment is to locate the center of mass of the "auto," using only spring-balance readings and length measurements.

Two sets of readings are necessary. For the first, adjust the lengths of the strings which hold the auto so that it is horizontal and adjust the supports so the strings are vertical. Refer to Figure 29. Measure L_1, the horizontal distance between the supporting strings, and record the readings of the spring balances. Call these readings F_A and F_B, respectively. The total weight of the object is $F_A + F_B$. (Why is this so?) This force of gravity W acts through the center of mass,

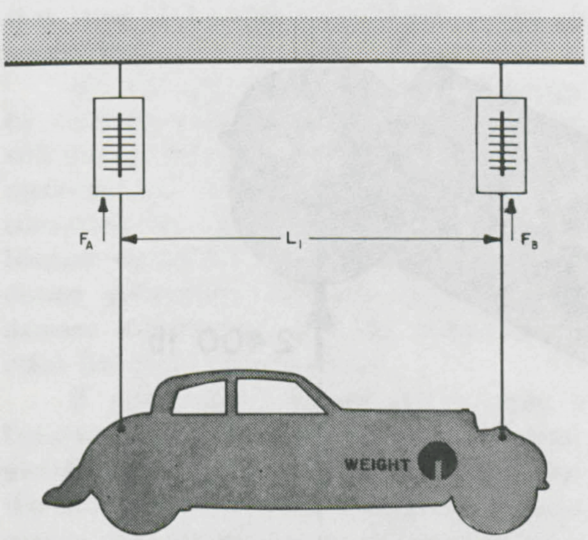

Figure 29.

which we assume is an unknown horizontal distance x_1 to the right of the left support point. Thus the torques about the left support point, which sum to zero, are

$$Wx_1 - F_B L_1 = 0$$

or

$$(F_A + F_B)x_1 - F_B L_1 = 0$$

Thus

$$x_1 = \frac{F_B}{F_A + F_B} L_1$$

Using your measured values, calculate the value of x_1. On the paper attached to the board, draw a vertical line a horizontal distance x_1 to the right of the left support point. The center of mass must lie on this line.

Now lengthen the support string at the rear of the auto so that the board tilts through an angle of at least 30°. Readjust the supporting strings so they are again vertical. The two strings will now be closer together (see Figure 30). Measure the new horizontal distance L_2. Read and record both balances again. Call these new readings on the left and right balances F_C and F_D, respectively. Assume that the center of mass is a horizontal distance x_2 to the right of the left support point. The torques about the left support point again sum to zero:

$$Wx_2 - F_D L_2 = 0$$

or

$$(F_C + F_D)x_2 - F_D L_2 = 0$$

Thus:

$$x_2 = \frac{F_D}{F_C + F_D} L_2$$

(In your experiment is $F_A + F_B = F_C + F_D$? Why should it be?)

On the paper attached to the board, draw a second vertical line a horizontal distance x_2 to the right of the left support point. The center of mass lies on this line and on the first line; it must therefore be at the point of intersection of the two lines.

Problem 6. An antique sedan standing on a horizontal road has 2260 lb of its weight supported by the front tires (1130 lb per tire) and 1440 lb supported by the rear tires. The distance between axles is 10 ft. It is then jacked up into the position shown in Figure 31, where the weight supported by the front tires is found to be 2400 lb. By drawing lines on Figure 31, which is drawn to the scale of 1 cm per ft, find the center of mass of the car.

Problem 7. Discuss the cartoon in Figure 32 in light of the principles of physics you have been studying. Is he really a practical person?

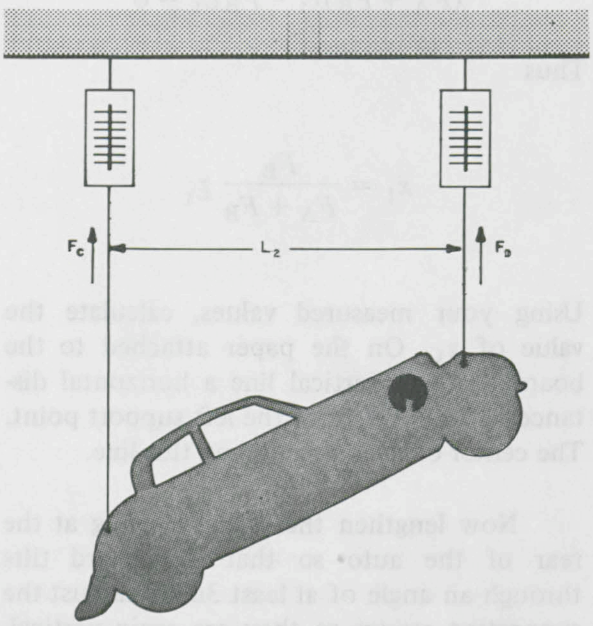

Figure 30.

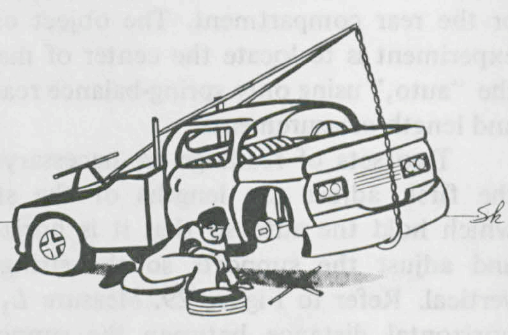

Figure 32. "I don't care about the laws of physics. I'm a practical man!"

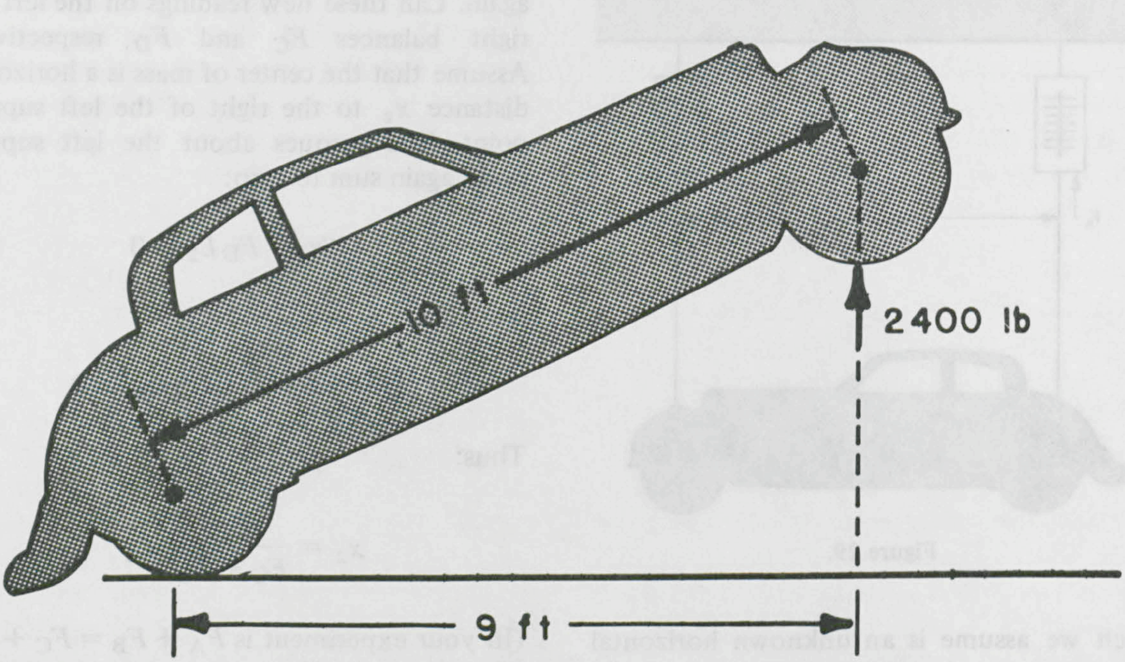

Figure 31.

ENERGY

Energy is one of the most useful concepts that you will encounter in your study of physics. Energy is a word that is much in the news today. We are familiar with the idea that the heat added to our homes to keep us warm in the winter is a form of energy. We also know that this energy existed in the coal or oil before it was burned to produce the heat. Or perhaps the energy was carried to our house by an electric current, and perhaps the power company created the electrical energy by using the energy of falling water. These possibilities suggest that energy exists in many forms, and that it can be converted from one form to another. This multiplicity of forms makes energy difficult to define.

It is known from experiments on energy conversions that energy is changed only in form, not in amount, during these conversions. It is this feature, known as the *Principle of Conservation of Energy,* that makes energy such a useful concept. Of course, we also speak of "wasting energy," which may suggest that the conservation law is not always obeyed. However, we do have evidence that when energy is changed from one form to others, the sum of all of the kinds of energy produced is just equal to the energy we started with. "Wasting" energy means only that some of it is converted to a form that we cannot use.

We will approach the subject of energy by defining a unit of *heat energy.* Then we will discuss a few other forms of energy and methods by which this energy could be converted to heat. This approach is used because we are interested in the damage done during automobile collisions, and permanent damage is associated with the conversion of other forms of energy to heat.

If you hold a lighted match under a beaker of water, the temperature of the water increases. This is an indirect indication that the energy of the water has changed. The heat energy thus transferred is measured in calories. If one gram of water has its temperature raised by one degree on the Celsius (centigrade) scale, one *calorie* (cal) of heat entered the water.*

Moving objects contain energy in a form called *kinetic energy.* If you are cruising along in your auto and suddenly put your foot on the brake, causing the car to skid to a stop, the kinetic energy disappears. The brakes, the tires, and the road become slightly warmer. A detailed analysis, as well as careful measurements on simple objects, leads to an exact definition of kinetic energy K:

$$K = (\tfrac{1}{2})mV^2 \qquad (6)$$

where m is the mass of the object and V is its velocity. When the mass is expressed in kilograms and the velocity in meters per second, the kinetic energy is in joules ($1\ \text{J} = \text{kg}\ (\text{m/s})^2$).

$$1\ \text{cal} = 4.186\ \text{J}$$

If you raise a car with a hydraulic lift, the car has energy in the form of *gravitational potential energy.* The lift that raises the car supplies the energy, which it gets from another source. If the car falls from its perch, its potential energy is converted first to kinetic energy and, during the crash, to heat energy, sound energy, and the energy required to bend metal.

If a car with a spring-mounted bumper moves at a very low velocity and collides with a wall, the springs compress as the car comes to rest. Energy is stored in temporarily compressed springs in a form called *elastic potential energy.* The amount of energy stored in the springs while the car is momentarily at rest before rebounding is equal to the original kinetic energy of the car. The greater the compression of the springs, the greater the amount of elastic potential energy. If the springs are *elastic,* they recover their original shape, pushing the car back into motion with the same speed in the opposite direction. In this case no kinetic energy is lost, no heat is

*The "calorie" you may count if you diet is actually a *kilocalorie*, or 1000 of these calories.

formed, and no permanent damage is done to the bumper. If the distortion of the springs exceeds an amount known as the *yield point,* the springs do not recover their original shape. Permanent damage is done, and some, or all, of the initial kinetic energy is converted to heat. (See Figure 33.) If you are not familiar with the production of heat by deformation, you may want to do a simple experiment. Bend a paper clip or other piece of wire rapidly back and forth several times. Place the bent part against your face. Has its temperature increased?

Example 5. A car is raised on a hydraulic lift so that work can be done on the radiator. Just then the bottom of the radiator falls off and 5 kg of water falls into a pail on the floor. A slightly weird physics teacher who is lounging around the gas station quickly measures the temperature of the water with a very sensitive thermometer. He finds that, in the process of falling into the pail, the water's temperature was raised 0.005°C.

a. How much heat energy, in calories, was required to produce this temperature increase?

b. What was the kinetic energy, in joules, of the water just before it hit the pail?

c. What was the velocity of the water just before it hit the pail?

d. How much gravitational potential energy with respect to the pail does the water have before the radiator is opened?

Solution.

a. Since 1 cal raises the temperature of 1 g of water by 1°C, we can write a formula for relating mass m, heat H, and temperature change ΔT:

$$H = (1 \text{ cal/g}^\circ\text{C}) m \Delta T \qquad (7)$$

The quantity in parentheses in this equation is just a constant which defines the calorie. Putting in the numbers:

$$H = (1 \text{ cal/g}^\circ\text{C}) \times 5000 \text{ g} \times 0.005^\circ\text{C}$$
$$= 25 \text{ cal}$$

b. This heat energy must have been supplied by the kinetic energy of the falling water. That is, if energy is conserved,

$$K \text{ (just before impact)} = H \text{ (after impact)}$$

$$K = 25 \text{ cal} \times 4.2 \text{ J/cal}$$
$$= 105 \text{ J}$$

c. We can use Equation (6) to find the velocity.

$$K = 105 \text{ J} = (\tfrac{1}{2}) m V^2$$

Solving for V,

$$V = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 105 \text{ kg}\cdot\text{m}^2/\text{s}^2}{5 \text{ kg}}}$$

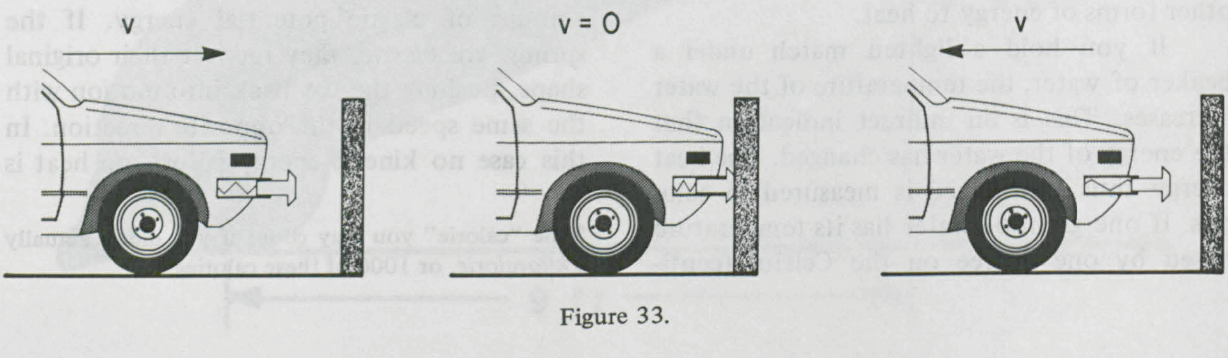

Figure 33.

d. If energy is conserved, this energy must have originally been stored in the water as gravitational potential energy. Thus the initial potential energy is 105 J.

Problem 8. A car with a mass of 1000 kg and a fancy elastic bumper is moving at 5 m/s. It strikes a wall and comes momentarily to rest without any permanent damage resulting from the collision. It then rebounds.

a. How much kinetic energy does the car have before the collision?

b. How much elastic potential energy is stored in the compressed bumper while the car is at rest?

c. How much kinetic energy does the car have after it rebounds from the collision?

SUMMARY

Average velocity (speed) is defined as the distance a body travels in a time interval divided by the time interval.

$$V_{\text{av}} = \frac{d}{t}$$

When the average velocity does not change from one time interval to the next, the motion is uniform.

When the average velocity *does* change from one time interval to the next, the motion is accelerated. If the change in velocity is the same during every time interval, the motion is uniformly accelerated. Then the acceleration is the change in velocity divided by the corresponding time increment. Mathematically:

$$a = \frac{\Delta V}{\Delta t}$$

Forces exerted on masses cause accelerations. The acceleration of a mass m produced by a total force F is given by

$$a = F/m$$

Every body has associated with it a point called the center of mass. That point moves just as if all the mass of the body were there and as if all the forces on the body acted at that point.

A force that does not act through the center of mass of an object produces a torque that tends to cause the body to rotate about its center of mass. If a body does not have a rotational acceleration, the sum of the torques about its center of mass equals zero.

SECTION B

Momentum and Collisions

THE TRANSFER OF MOTION THROUGH SIMPLE COLLISIONS

If a massive moving object collides with another object, the motion is not stopped immediately. Either the wreckage moves after the collision, or the objects rebound. Also, when two objects initially at rest separate by pushing against each other, they acquire motions in opposite directions. A thorough understanding of these phenomena is essential if we are to discuss automobile collisions meaningfully. Let us study such collisions and separations with some care.

EXPERIMENT B-1. Collisions with Carts

To perform this experiment you will need a cart with a low, flat surface and large wheels with good bearings. The kind, like the one in Figure 34, used to move heavy cartons from one place to another within a warehouse works well. You will also need a large, level floor area, such as a hallway or a gymnasium. The experiment can be performed either by small groups of students, each group with its own cart or its own turn at using the cart, or as a class experiment, with a few persons at a time actively carrying out the various steps and the others observing and making suggestions. The persons jumping on and off carts should take care not to become casualties.

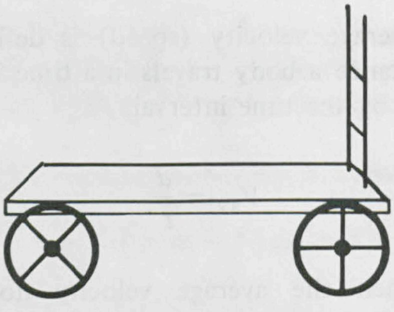

Figure 34.

1. The cart is initially stationary and empty. One small person should run slowly and jump on the cart. Measure how far the cart rolls before it stops.

2. The cart is initially stationary and empty. The same person should run more rapidly than in step 1 and jump on the cart. Measure how far the cart rolls before it stops. How do the results of steps 1 and 2 compare?

3. The cart is initially stationary and empty. A heavier (more massive) person should run at about the same velocity as did the person in step 1 and jump on the cart. Measure how far the cart rolls before it stops. How do the results of steps 1 and 3 compare?

4. The cart is initially stationary and the heavier person is standing on it. The lighter person should run at as nearly the same speed as in step 1 as possible and jump on. Measure how far the cart rolls before it stops. How do the results of steps 1 and 4 compare?

5. The cart is initially stationary, and the same person as in step 4 (or one with a similar mass) is standing on it. The mass and velocity of the jumper should duplicate step 4 as much as possible, but this time the jumper should jump on the back edge and right off again in a backward direction. Measure how far the cart rolls before it stops. How do the results of steps 4 and 5 compare?

The remaining steps will call for persons to jump from a stationary cart. In each case comparisons between two situations are to be made.

6. Two persons are on the cart. Compare the motion of the cart after one jumps off with the motion after both jump off in the *same* direction simultaneously.

7. Two persons are initially on the cart. Compare the motion of the cart after they both jump off simultaneously in the same direction with the motion after they both jump off simultaneously in opposite directions.

8. One person is initially on the cart. Compare the motion of the cart after he or she jumps off trying to jump as high as possible with the motion of the cart after he or she jumps off trying to jump as far as possible.

9. One person is initially standing on the cart at one end. The person should take quick steps toward the other end, but stop before going off the cart. How does the cart move?

MOMENTUM

The momentum of a moving object, such as an automobile, is defined as the mass of the object times its velocity:

$$p = mV \qquad (8)$$

Like velocity, momentum is a vector; its direction is the same as that of the velocity. The concept of momentum is particularly useful in analyzing collisions. As you will see, during a collision the total momentum of an isolated system stays constant. This allows prediction of some of the results of collisions, even without a detailed knowledge of the forces acting during the collision. We will first develop this idea in mathematical form; then we will apply it to simulated auto collisions.

Consider a collision between two autos. During the collision, the velocity of each car changes; that is, each car experiences an acceleration. You know from Newton's second law (Equation 4) that the acceleration of each car is determined by the forces acting on that car. If we assume that neither car leaves the surface of the road, we need concern ourselves only with forces parallel to the road. While these forces may include such things as friction between the tires and the road, the forces exerted on each car by the other during collision are by far the greatest forces involved. Thus, only the collision forces need be considered in determining the change of velocity (or change of momentum) that occurs during the collision.

Combining the definition of acceleration (Equation 3) and Newton's second law of motion (Equation 4), we can derive an equation for the change in momentum Δp of either car in terms of the collision force F exerted on it and the duration of the collision Δt.

For car #1:

$$F_1 = m_1 a_1 = m_1 \frac{\Delta V_1}{\Delta t_1}$$

The momentum change of car #1 (Δp_1) is the difference between its momentum after the collision ($p_{1\mathrm{f}}$) and its momentum before the collision ($p_{1\mathrm{i}}$). Mathematically:

$$\Delta p_1 = p_{1\mathrm{f}} - p_{1\mathrm{i}}$$

$$= m_1 V_{1\mathrm{f}} - m_1 V_{1\mathrm{i}}$$

$$= m_1 (V_{1\mathrm{f}} - V_{1\mathrm{i}}) = m_1 \Delta V_1$$

Thus

$$F_1 = \frac{\Delta p_1}{\Delta t}$$

Likewise, for car #2:

$$F_2 = m_2 a_2 = m_2 \frac{\Delta V_2}{\Delta t} = \frac{\Delta p_2}{\Delta t}$$

The force F_1 is the force which causes the momentum change (Δp_1) of car #1. It is the force exerted *by* car #2 *on* car #1. Similarly, F_2 is the force exerted *on* car #2 *by* car #1. Remember that Newton's third law states that for every action force (such as F_1) there is an equal and opposite reaction force (F_2). Thus we can assert that $F_1 = -F_2$. The minus sign means only that F_1 and F_2 are in opposite directions. (See Figure 35.) If we now add the equations for F_1 and F_2, we obtain

$$F_1 + F_2 = 0 = \frac{\Delta p_1}{\Delta t} + \frac{\Delta p_2}{\Delta t}$$

Multiplying by Δt, we have

$$\Delta p_1 + \Delta p_2 = 0$$

This equation says that the change in momentum of car #1 (Δp_1) is equal and opposite to the change in momentum of car #2 (Δp_2). Now the *total* momentum of this two-car system is just the sum $p = p_1 + p_2$. The *change* in total momentum as a result of the collision is

$$\Delta p = p_{\mathrm{f}} - p_{\mathrm{i}} = (p_{1\mathrm{f}} + p_{2\mathrm{f}}) - (p_{1\mathrm{i}} + p_{2\mathrm{i}})$$

$$= (p_{1\mathrm{f}} - p_{1\mathrm{i}}) + (p_{2\mathrm{f}} - p_{2\mathrm{i}})$$

$$= \Delta p_1 + \Delta p_2$$

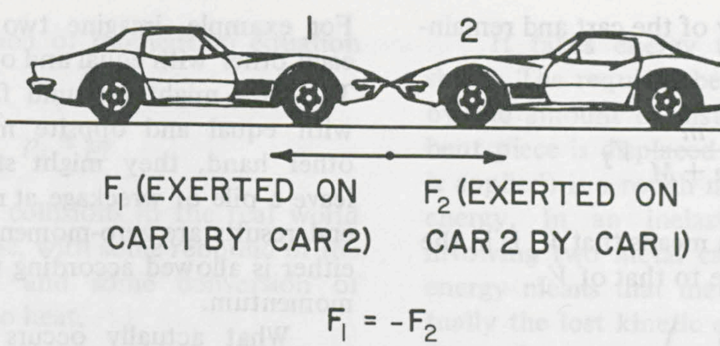

Figure 35.

But we have just shown that $\Delta p_1 + \Delta p_2 = 0$. Thus $\Delta p = 0$ and total momentum does not change as a result of the collision! This result is known as the *Law of Conservation of Momentum.* We can write this law in three equivalent forms. First, as we derived, if no net force acts on the system from the outside, the change in total momentum is zero:

$$\Delta p = 0$$

An equivalent way of saying this is to say that the momentum is constant throughout the collision. That is,

$$p = \text{constant}$$

A third way of saying the same thing is to say that the momentum before the collision is the same as the momentum after the collision, or

$$p_i = p_f$$

Example 6. In the part of Experiment B-1 in which two persons jump off the initially stationary cart, assume that the mass of each person is $m = 72$ kg and that the mass of the cart is $M = 36$ kg. Assume also that when either jumps off, he acquires a velocity of 4 m/s.

a. Starting with both persons on the cart, how fast is the cart moving immediately after one person jumps off?

b. How fast will the cart be moving if both persons jump off simultaneously?

Solution.

a. Represent the mass of a jumper by m and the mass of the cart by M. Before jumping off, both the jumpers and the cart are at rest and have zero momentum. Thus the total initial momentum p_i of the system is zero. After jumping off, the jumper has velocity $V_J = -4$ m/s. The minus sign means that the jumper is moving to the left (Figure 36). The momentum of the jumper is $m \cdot V_J$ (to the left). The momentum of the cart (plus the other jumper) is $(m + M)V_c$.

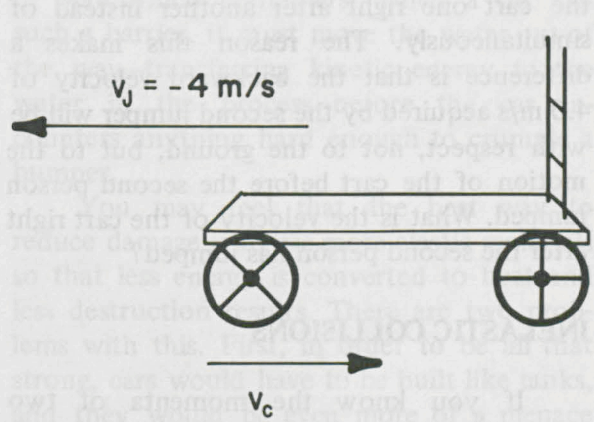

Figure 36.

Since the momentum after jumping must be the same as the initial momentum, we conclude that

$$p_i = 0 = p_f = mV_J + (m + M)V_c$$

Then the velocity of the cart and remaining jumper is

$$V_c = -\frac{m}{m+M} V_J$$

The negative sign means that V_c is in the direction opposite to that of V_J.

$$V_c = -\left(\frac{72}{72+36}\right) (-4 \text{ m/s})$$

$$= +2.7 \text{ m/s (to the right)}$$

b. $p_i = 0 = p_f = 2\,mV_J + MV_c$

$$V_c = -\frac{2\,m}{M} V_J = \frac{2 \cdot 72}{36} (4 \text{ m/s})$$

$$= 16 \text{ m/s (to the right)}$$

Analysis: Using the same ideas as in Example 6, analyze the results of Experiment B-1 to see if your results agree with what you would expect from momentum conservation. What effect would you expect frictional forces to have?

Problem 9. Solve Example 6b again, but this time assume that the two persons jump off the cart one right after another instead of simultaneously. The reason this makes a difference is that the backward velocity of 4.0 m/s acquired by the second jumper will be with respect, not to the ground, but to the motion of the cart before the second person jumped. What is the velocity of the cart right after the second person has jumped?

INELASTIC COLLISIONS

If you know the momenta of two automobiles before they collide, you can add them and thus find the total momentum of the two. This will be the total momentum both before and after the collision. However, without additional information, you cannot determine how the total momentum is divided between the two cars after the collision. For example, imagine two cars approaching each other with equal and opposite momenta. The cars might rebound from one another with equal and oppsite momenta. On the other hand, they might stick together and leave a pile of wreckage at rest. Both of these end results are zero-momentum systems, and either is allowed according to conservation of momentum.

What actually occurs depends on the *elasticity* of the bumpers and other parts involved in the collision. If the bumpers are perfectly elastic, they spring back to their original shape after the collision. If they are highly *inelastic,* they deform and collapse without any sign of going back to their original shapes. Which of these possibilities happens depends on properties of the materials as well as on the speeds and masses of the cars. One measure of how elastic the vehicles are is the amount of energy that is converted from kinetic energy before the collision to heat energy after the collision. However, not only is it difficult to measure heat energy generated in processes such as this, but luckily it is unnecessary. Based on many experiments, we have good reason to believe that energy is never destroyed; it only changes form. Thus we need only to measure kinetic energy before and after the collision; the difference in the two measurements must appear in some other form, such as heat energy, generated during the collision.

PERFECTLY ELASTIC COLLISIONS

In a *perfectly elastic collision,* no kinetic energy is lost. That is, the total kinetic energy after the collision is the same as the kinetic energy before the collision. This allows the writing of an equation for conservation of kinetic energy:

$$K_i = K_f$$

where the subscript i refers to initial conditions (just before the collision) and f to final conditions (just after the collision). In every

case, a conservation of momentum equation holds:

$$p_i = p_f$$

Very many collisions in the real world are partially elastic, with some rebound of the colliding objects and some conversion of kinetic energy into heat.

COMPLETELY INELASTIC COLLISIONS

The third and final case is the *completely inelastic collision.* This occurs when the two objects stick together during the collision and move as one afterwards. Since this is a very common case for automobile collisions and can be easily solved, we shall examine it more closely for head-on collisions. The basic equation is again that for conservation of momentum, $p_i = p_f$. If the two objects are labeled with the subscripts *1* and *2*, and the final velocity of the two, stuck together, is V_f, this gives:

$$m_1 V_1 + m_2 V_2 = (m_1 + m_2)V_f \qquad (9)$$

where V_1 and V_2 are the velocities before the collision.

The amount of damage to the cars depends largely on how much energy is converted from kinetic to heat energy. Thus a measure of the damage is the loss of kinetic energy ΔK. This quantity can be computed from the equation

$$\Delta K = K_i - K_f$$

$$= \left(\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2\right) - \frac{1}{2}(m_1 + m_2)V_f^2$$

The initial velocities, V_1 and V_2, may be measured and V_f may be computed from Equation (9).

It takes energy to bend metal out of shape. The required bending force multiplied by the amount of distortion (the distance a bent piece is displaced at the point the force is applied) is a rough measure of the required energy. In an inelastic, head-on collision involving two metal cars, the loss of kinetic energy means that metal will be bent. Eventually the lost kinetic energy shows up in the form of temperature increases associated with the permanent deformations of the solid structures. If the metal is strong—that is, if large forces are required to bend it—the amount of deformation may be slight. If the metal is weak, large deformations may occur. (However, minimizing damage to the car does not necessarily maximize passenger safety.)

Of course, there are other forms into which the lost kinetic energy may be changed. If the collision is not head-on, some rotations will be introduced, and these involve rotational kinetic energies. Other things being equal, collisions which result in rotations produce less damage, provided the cars come to rest without additional collisions with other objects. Another way to reduce damage is to get rid of some energy by transferring it to objects other than the cars. This is the principle behind the use of barriers consisting of water-filled containers. When a car hits such a barrier, it must move the water out of the way—transferring kinetic energy to the water in the process—before the car encounters anything hard enough to crumple a bumper.

You may feel that the best way to reduce damage is to use more elastic materials so that less energy is converted to heat and less destruction results. There are two problems with this. First, in order to be all that strong, cars would have to be built like tanks, and they would be even more of a menace than they now are. Secondly, in an elastic collision, the car would rebound and smash the occupants even more. The inelastic collision is terrible for both the car and its occupants; the elastic collision might cause less damage to the car, but it would be even worse for the occupants.

Anyway, even highly elastic materials have an elastic limit. A large enough deforming force will cause a permanent deformation regardless of how well the object restores itself after release from smaller forces. The forces that occur during high-speed collisions of massive objects, such as automobiles, always exceed the elastic limits of their metal structures.

Example 7. In one part of Experiment B-1, a person ran and jumped onto the stationary cart which already had a second person standing on it. Take the mass of each person to be $m = 72$ kg and the mass of the cart as $M = 36$ kg, and assume that the first person is moving with a speed of 5.0 m/s as he jumps on the cart.

a. Is the "collision" elastic or inelastic?

b. What are the momentum and kinetic energy of the person jumping before he jumps on the cart?

c. Immediately afterward, what is the momentum of the cart and its occupants? Its velocity? Its kinetic energy?

d. How much heat energy (in joules) is produced by the collision?

e. How much heat energy is dissipated in friction as the cart rolls to a stop?

Solution.

a. The collision is inelastic, since the two objects stick together afterwards. We then know that although momentum is conserved, kinetic energy is not.

b. The initial momentum and energy are just those of the runner:

$$p_i = mV_i = 72 \text{ kg} \times 5.0 \text{ m/s} = 360 \text{ kg·m/s}$$

$$K_i = \frac{1}{2}\,mV_i^2 = \frac{1}{2} \times 72 \text{ kg} \times 25 \text{ m}^2/\text{s}^2$$

$$= 900 \text{ J}$$

c. By conservation of momentum, the final momentum (cart and occupants) is the same as the initial momentum:

$$p_f = (2\,m + M)V_f = 360 \text{ kg·m/s}$$

Then:

$$V_f = \frac{p_f}{(2\,m + M)} = \frac{360 \text{ kg·m/s}}{(2 \times 72 + 36) \text{ kg}} = 2.0 \text{ m/s}$$

$$K_i = \frac{1}{2}\,(2\,m + M)V_f^2 = \frac{1}{2} \times 180 \text{ kg} \times 4 \text{ m}^2/\text{s}^2$$

$$= 360 \text{ J}$$

d. Heat $= \Delta K = K_i - K_f$

$$= 900 \text{ J} - 360 \text{ J} = 540 \text{ J}$$

e. All of the kinetic energy which the cart had right after the collision is lost to friction. That is,

$$\text{Heat} = K_f = 360 \text{ J}$$

Problem 10. In step 5 of Experiment B-1, a person jumps onto the back edge of the cart and immediately back off, while a second person stands on the cart. Assume that the person who jumps has a velocity of 4.5 m/s forward as he jumps on and 5.0 m/s backward (with respect to the floor) as he jumps off. As before, take $m = 72$ kg and $M = 36$ kg.

a. What are the momentum and kinetic energy of the person jumping just before he jumps on the cart?

b. What are his momentum and kinetic energy after he jumps off the cart?

c. What is the momentum of the cart plus rider after the jumper jumps off the cart? The velocity? The kinetic energy?

d. How much energy does the system (cart plus two persons) gain (or lose) during the jumping on and jumping off processes? Where does this energy come from (or go to)?

EXPERIMENT B-2. Momentum Conservation and Energy Loss in Simulated Collisions

In this experiment you will have an opportunity to view a film that shows several *simulated* collisions between a car and a truck. You will make measurements of the velocities of both vehicles before and after each collision. The velocities are constant (except during the collisions) so all you need to do is record the position of each vehicle at two different times and calculate their velocities using Equation (1), which we will rewrite in the form

$$V = \frac{\Delta x}{\Delta t}$$

In this equation, Δx is the difference between two positions of the car or truck along a road, and Δt is the corresponding time interval. When you record data, each x will represent a number that is associated with the position of a vehicle on the road, and the corresponding t is the reading of the clock. Each reading should be made and recorded while the film is stopped, so that you can perform these operations with care.

To associate numbers with positions, you need some information about the lengths of road markers. The white stripes down the center of the road are each 10.0 m long, and the distances between stripes are also 10.0 m. Try to read the positions you record to the nearest 1.0 m. You will find it helpful to fix your attention on a particular point on a vehicle, such as its front bumper, whenever you record its position.

You will be able to determine time easily because a clock in each picture indicates the time in seconds. Read the clock to the nearest 0.1 s. Each time the film indicates a stop frame, stop the projector so you can read positions and times at your leisure.

To calculate momenta, you need to know that the mass of the car is 2000 kg and the mass of the truck is 4000 kg. Let motion to the right correspond to a positive momentum and motion to the left to a negative momentum.

Finally, it will be interesting to find out how much kinetic energy is lost in each collision. The result is the amount of energy converted to heat (and thus involved in doing damage) during the collision.

PART 1

The film shows a moving car striking a stationary truck. Measure the velocity of the car before the collision and that of the wreckage after the collision. Use these velocities to calculate momenta and kinetic energy before and after the collision, and complete Table I on the data page at the back of the module.

Question 1. Is the total momentum of the system after the collision the same as the total momentum before the collision (to within the accuracy of your measurements)?

Question 2. How much kinetic energy is converted to heat during the collision? What fraction of the initial kinetic energy is this?

PART 2

In the second collision on the film, the truck is moving, but the car overtakes it and the two vehicles collide and lock together. There is an extra velocity to measure, but no new problems are presented. Complete Table II on the data page.

Question 3. Is the total momentum of the system conserved in this case?

Question 4. Is the fraction of the total kinetic energy which is lost during this collision higher or lower than that calculated in Question 2?

PART 3

In the third collision, the car and the truck are approaching each other, and the wreckage is at rest after the collision. That is, the velocity after the collision is zero. Remember to be careful about the directions of motion and thus the sign of each individual

momentum when finding the total momentum of the system before the collision. Complete Table III on the data page.

Question 5. The momentum of the system after the collision is clearly zero. Is it also zero before the collision?

Question 6. What fraction of the initial kinetic energy of the system is lost during the collision?

PART 4 (Optional)

In Part 3 you viewed a collision between two vehicles which had a total momentum of zero. After the collision, the wreckage was at rest. Thus, the center of mass of the wreckage is at rest. Remember that the center of mass of a system is the point that behaves as if all the mass of the system were concentrated there and as if all the forces acting on the system act there.* Since the truck and the car exert equal and opposite forces on each other during the collision (and since the much smaller horizontal forces supplied by the road are not taken into account), there is no net force on this system to cause the center of mass to change its velocity. If the center of mass is at rest after the collision, it is also at rest before the collision. If you were at the center of mass before the collision (not a wise procedure!), the truck and the car would appear to approach you from opposite directions and with equal momenta.

Now think back to Part 1. The center of mass of the car-truck system before the collision was somewhere between the car and the truck. The truck was at rest but the car moved to the right, so the center of mass also moved to the right, remaining between the two vehicles. If you had been able to move with the center of mass, what would you have seen? Watch the next collision on the film to find out.

Make measurements for this case as before, and fill in Table IV with the results of your calculations.

*** See the Appendix.**

Question 7. What is the momentum of each vehicle when the system is viewed from the center of mass? What is the total momentum of the system?

Question 8. How much kinetic energy is converted to heat in this collision? Is the result the same in Part 4 as in Part 1? Should it be?

The next collision on the film is a repeat of the collision in Part 2, as viewed from the center of mass. You need not repeat the measurements for this case, but you may if you wish.

PART 5 (Optional)

In the final collision, the car and truck approach each other at right angles. Measurements of the velocities of car and truck before the collision and the wreckage after the collision may be done as before, and Table V may be completed. The momentum associated with each of these velocities can also be found as before. However, the addition of the two momenta at right angles must be carried out according to the rules of *vector addition.* We will use a simple *displacement* vector to illustrate how to add vectors.

If you walk 3 m east and then walk 4 m north, how far would you be from where you started? You can find the answer by using either the Pythagorean Theorem or a graphical method. We will illustrate the latter. Refer to Figure 37. Draw an arrow to the right (east) of length proportional to the distance 3 m. (We have made it 3 cm long.) From the tip of this first arrow, draw a second arrow toward the top of the page (north) corresponding to a displacement of 4 m (in this case, 4 cm). Now draw the *resultant* arrow from the base of the first to the tip of the second. The length of the resultant represents the distance we are seeking. The direction of the resultant arrow also indicates the direction you would be from

your starting point after walking both east and north.

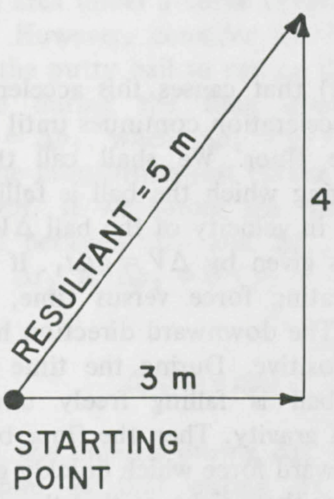

Figure 37. The addition of vectors.

Now refer to the data you took from the film of the two-dimensional collision (Table V). To find the momentum of the system before the collision, combine the two momenta in the same way in which displacements were combined in Figure 37. Compare the resultant with the measured momentum of the system after the collision.

In principle, it is possible to compare the angle the resultant on your drawing makes with the east-west direction and the angle the diagonal road makes with the horizontal road in the film. However, since the camera was not held directly above the collision area, angles appear distorted in the film. Those who like problems in geometry can unravel the distortion and make the comparison by knowing that the clock, which looks elliptical in the film, is really circular.

SUMMARY

Jumping on and off a cart causes the cart to change its velocity in a direction opposite to the change in velocity of the jumper. The magnitude of these effects depends on the masses of the cart and the jumper.

The momentum of an object is its mass times its velocity:

$$p = mV$$

When two objects collide, whether the collision is elastic or inelastic, the total momentum of the system does not change as long as there is no net force from outside the system.

When two bodies collide inelastically, kinetic energy is lost and heat energy is produced.

When two bodies collide in a totally inelastic manner, the final velocity of the two (locked together) is

$$V_f = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

where V_1 and V_2 are the velocities of m_1 and m_2, respectively, before the collision. V_1 and V_2 may be either positive or negative quantities, depending on their directions.

The heat energy produced in a collision is given by

$$\text{Heat energy} = K_i - K_f$$

The kinetic energy of each object is given by

$$K = (1/2)mV^2$$

In a zero-momentum totally inelastic collision, 100% of the kinetic energy is lost.

SECTION C

Forces

THE IMPULSE-MOMENTUM THEOREM

In an earlier section, we wrote Newton's second law in a form which related the force on an object to the rate at which its momentum changes. That relation is:

$$F = \frac{\Delta p}{\Delta t} \qquad (10)$$

where Δp is the change in momentum and Δt is the time during which the force acts. Earlier we were primarily interested in the initial and final motions of two colliding bodies. We deliberately ignored details of the collision, such as values of F and Δt, that produce a particular momentum change in a single body. However, if we want to study either the nature of the damage done to an auto during a collision or methods of minimizing injury to passengers, the forces that bring the moving bodies to rest are all-important. A moving auto (or any passenger) has an initial momentum ($p = mV$) that must be reduced to zero when the auto stops. According to Equation (10), the only way this can be done is to exert a total force F for a time Δt, such that the product of $F{\cdot}\Delta t$ is equal to the required change in momentum. This product is called the *impulse* on the body. We rewrite Equation (10) in a form known as the *impulse-momentum theorem*:

Impulse = change in momentum

or

$$F{\cdot}\Delta t = \Delta p \qquad (11)$$

Consider a simple example. If you drop a putty ball from some height above the floor, it falls freely toward the floor under the influence of gravity. Since its acceleration is $g(= 9.8\ \mathrm{m/s^2})$, the constant force of gravity (the weight) that causes this acceleration is *mg.* This acceleration continues until the ball reaches the floor. We shall call the time interval during which the ball is falling Δt_1. The change in velocity of the ball ΔV during that time is given by $\Delta V = g\Delta t_1$. If we plot the accelerating force versus time, we get Figure 38. The downward direction has been taken as positive. During the time interval Δt_1, the ball is falling freely under the influence of gravity. Then the floor begins to exert an upward force which quickly grows to be as great as the weight, so that the net force is zero. Then, during the time interval Δt_2, the net force is upward and the ball is slowing down. Finally, the upward force of the floor again equals the weight, and the putty ball has come to rest.

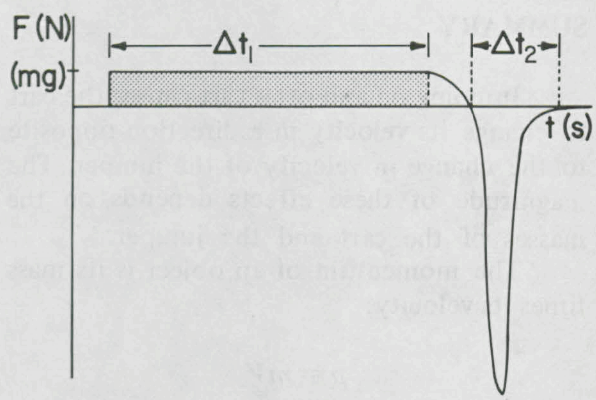

Figure 38. The net force acting on a ball of putty.

Note that, as with all inelastic collisions, kinetic energy is changed into heat energy during the collision.

Consider the interval Δt_1 on the graph. In this interval, the area of the rectangle between the graph and the t axis is $F_1\Delta t_1$. But this is just the impulse! In fact, a detailed analysis would show that the area between a curve and the t axis on an F versus t graph is *always* an impulse. Thus, for an *F-t* graph, the so-called "area under a curve" is a measure of

the impulse and of the change of momentum.

If all *F-t* graphs were straight lines and the enclosed areas were rectangles, the concept of area under a curve would not be very useful. However, consider the impulse that brings the putty ball to rest on the floor. The impulse during the time interval Δt_1 is $F_1 \Delta t_1 = \Delta p_1$, where $\Delta p_1 = p_f - 0 = p_f$, and p_f is the momentum just before the ball hits the floor. If we ignore the very tiny time interval between Δt_1 and Δt_2, the impulse during Δt_2 is $\Delta p_2 = 0 - p_f = -p_f = -\Delta p_1$. Thus

$$\Delta p_2 = -\Delta p_1$$

That is, the impulse during Δt_2 has the same magnitude as that during Δt_1, but it is negative (oppositely directed). Also, the stopping time Δt_2 is much smaller than Δt_1. It is so small that some sophisticated instrument must be used to measure it. In addition, the small value of Δt_2 implies a large average decelerating force $(F_2)_{av}$. Not only is this force hard to measure because of its large magnitude and short duration, it is not constant over the interval Δt_2; it builds up and collapses (in a direction opposite to gravity) in a manner similar to that shown in Figure 38. Since this impulse is the same as that during Δt_1, the *area* "under" the *F-t* curve during Δt_2 must be equal to the area of the rectangle. Such an area below the horizontal axis is said to be negative; thus this second impulse is oppositely directed (upward).

The damage done to an automobile and to a passenger during a collision is largely determined by the sizes of the forces that bring them to rest. It is important to discover how to measure these forces and to learn what factors influence them. Experiment C-1 will provide an opportunity to explore these matters. Note in particular that, in each case, a particular change of momentum must be produced to bring the object to rest; anything that lengthens the time of deceleration will help to reduce the damage-producing forces.

Two other factors strongly influence the actual damage done by given forces. One is the area over which the forces are distributed. The amount of deformation of a rigid body such as a bumper (or a skull) is determined by the total applied force divided by the area to which it is applied, a quantity called *stress:**

$$\text{Stress} = \frac{\text{force}}{\text{area}} \tag{12}$$

Another way to minimize damage to a surface during a collision is to minimize the stress by distributing the decelerating forces over as large an area as possible.

The other factor of importance is the "strength" of the object to which the stress is applied. A measure of this strength is *strain*:

$$\text{Strain} = \frac{\text{change of length under stress}}{\text{original length}}$$

As long as the applied stress is small enough, the resulting strain is proportional to the stress for many materials:

$$\text{Stress} = k \times \text{strain} \tag{13}$$

If the material is strong, in the sense that it is hard to compress or bend, the constant k is large. As long as Equation (13) holds, no permanent damage is done. The original energy that caused the deformations is stored as elastic potential energy, and it is soon recovered as kinetic energy when the object springs back to its original shape.

However, for high-speed automobile collisions, some of the parts involved are permanently deformed. For a given object, the stress at which permanent deformation begins is called the *elastic limit.* The important factor in determining which parts are damaged is the elastic limit of each part.

Example 8. A car (mass 1000 kg) with a "springy" bumper approaches a stationary barrier at a constant speed. While the bumper is in contact with the barrier, the force of interaction between them increases as the

*A stress which compresses a fluid is usually called *pressure.*

bumper bends to its maximum deformation and then decreases as the car rebounds. The graph of force versus time is shown in Figure 39.

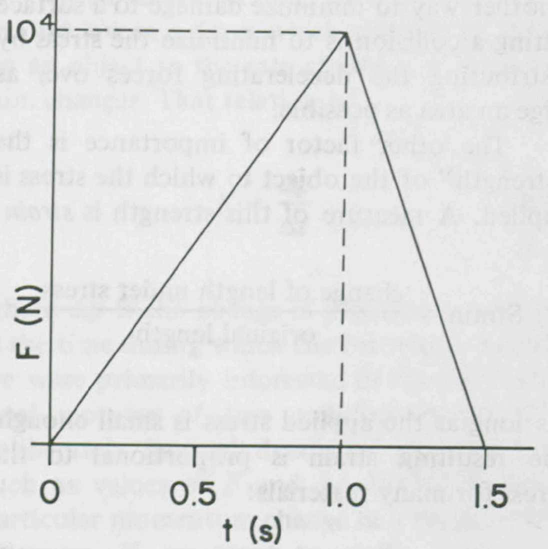

Figure 39.

a. What was the velocity (in m/s) of the car before it struck the barrier?

b. What was the velocity of the car after it rebounded from the barrier?

c. What was the average force during the time of contact between bumper and barrier?

d. If the area of contact was 0.5 m^2, what maximum stress did the bumper exert on the barrier?

e. Did the stress on the bumper ever exceed the elastic limit of the bumper?

f. How much heat energy (in joules), if any, was generated during this collision?

Solution.

a. The area under the rising portion of the F versus t curve is equal to the impulse caused by the forces which stop the car (Figure 39). Since this part of the area is a right triangle, we can calculate it by using the formula for the area of a triangle. That is, the area equals

$$\tfrac{1}{2}(F_{max})\Delta t = 5000 \text{ N·s}$$

This is equal to the change in momentum of the car as it is brought to rest. That is

$$\Delta p = mV_f - mV_i = 0 - mV_i$$

$$= -mV_i$$

or

$$V_i = -\frac{\Delta p}{m} = -\frac{5000 \text{ N·s}}{1000 \text{ kg}} = -5 \text{ m/s}$$

The negative sign indicates that the initial velocity is in the opposite direction from that of the force, which we have chosen to be positive.

b. The area under the falling part of the F versus t curve is the impulse given to the car as it rebounds. This area is half as large as that calculated above, or 2500 N·s. The final velocity is thus half as large, and oppositely directed to the initial velocity. That is, $V_f = +2.5$ m/s.

c. The average force F_{av} is defined as the force which, when multiplied by the entire time of interaction (1.5 s), equals the total impulse (7500 N·s). Thus,

$$F_{av} \cdot \Delta t = \Delta p$$

$$F_{av} = \frac{7500 \text{ N·s}}{1.5 \text{ s}} = 5000 \text{ N}$$

d. $$\text{Stress}_{max} = \frac{F_{max}}{A} = \frac{10{,}000 \text{ N}}{0.5 \text{ m}^2}$$

$$= 2 \times 10^4 \text{ N/m}^2$$

e. Yes, the elastic limit was exceeded. Otherwise the spring would have stored

and returned all of the initial kinetic energy, and the final velocity would have been equal in size to the initial velocity. Also, the *F-t* graph would have been "symmetrical."

f. Heat energy $= \Delta K = \frac{1}{2} m V_i^2 - \frac{1}{2} m V_f^2$

$$= \frac{1}{2} \times 1000 \text{ kg}$$

$$\times (5^2 - 2.5^2) \text{ m}^2/\text{s}^2$$

$$= 9375 \text{ J}$$

Problem 11. A 0.18-kg golf ball is driven at a velocity of 50 m/s by the head of a two wood. A high-speed motion picture shows that the ball is in contact with the head of the club for only 0.045 s.

a. What total impulse does the ball receive from the club?

b. What is the average force between the ball and the club while they are in contact?

EXPERIMENT C-1. Simulated Collisions

This experiment is performed with an air track and gliders (see Figure 40). One glider has an attachment that consists of a sleeve loaded with a spring and a plunger. The plunger has a flat surface covered by a piece of *Velcro* that will adhere to another piece of Velcro whenever the two pieces collide. When such a collision takes place, the plunger compresses the spring by an amount that is proportional to the maximum force of compression. A spring-loaded rachet on the sleeve locks the plunger in the sleeve at its position of maximum penetration. By measuring this maximum penetration, one can determine the maximum force of interaction between the colliding bodies.

PART 1

In this part of the experiment, a single glider will be released from different positions along a tilted air track. When the glider reaches the lower end of the track, it collides inelastically with a flat, immovable vertical surface at the end of the track. Be sure that there is a piece of Velcro on this surface at such a location that it will be struck by the Velcro on the plunger head. The velocity, just before the collision, can be calculated from a measurement of the vertical distance the glider travels down the track before collision by using the following equation:

$$V = \sqrt{2gh} \tag{14}$$

In this equation, g is the acceleration of gravity, 9.8 m/s^2, and h is the vertical distance traveled, as shown in Figure 41. The momentum of the glider just before collision is calculated by multiplying the velocity determined from Equation (14) by the mass of the glider.

$$p = mV = m \cdot \sqrt{2gh} \tag{15}$$

You will need to record the length of plunger sticking out of the sleeve before and after each collision. The difference d between these two measurements is the maximum compression of the spring during the collision. Once this is known, you can calculate the maximum force of compression F_{max} by multiplying d by a constant for that spring called the spring constant k. The average force F_{av} during the collision is about half the maximum force.

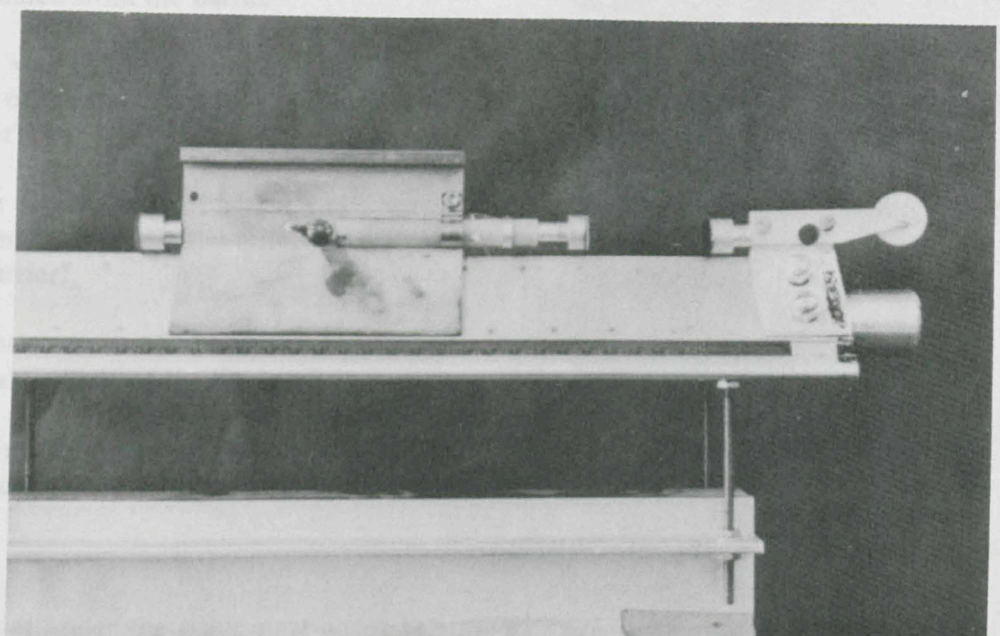

Figure 40.

$$F_{av} \approx (1/2)F_{max} = (1/2)kd \quad (16)$$

1. To determine k, turn the cart on end so that the spring is vertical with the plunger face up. Put a known mass M (perhaps a kilogram or so) on the plunger. Measure the amount d by which the spring compresses by measuring the plunger height before and after adding the load M. The spring constant (in N/m) can then be calculated from the equation:

$$k = F/d$$

$$= Mg/d$$

where M is in kilograms, $g = 9.8\ \mathrm{m/s^2}$, and d is in meters. Record your value for k.

2. Insert the plunger into the sleeve attached to your glider so that it rests against the spring but does not compress it. Be sure you can reproduce this setting in later steps. Record the length of plunger sticking out of the sleeve.

3. Place the glider about one-fourth of the way up the air track. Measure the height above the table of a reference mark placed somewhere on the glider. Move the glider to the other end of the air track and measure the height of the same reference mark. The difference in these two measurements is h. Release the glider from the first point (about one-fourth of the way up the track).

4. Use $p = m\sqrt{2gh}$ to find p just before impact.

5. After the collision, measure the length of the plunger sticking out of the sleeve. From this result and that of step 2, determine the maximum compression of the spring d.

6. Use $F_{av} = (1/2)kd$ to calculate F_{av} from the values of k and d determined in steps 1 and 5.

7. Use $F_{av}\Delta t = \Delta p$ to calculate the time of collision from the values of p and F_{av} determined in steps 4 and 6.

8. Repeat steps 2 through 7, releasing the glider from about halfway up the track. Since the new distance s is twice the old, the new momentum p should be $\sqrt{2}$ times that calculated in step 4.

9. Repeat steps 2 through 7 once more, using the full length of the track.

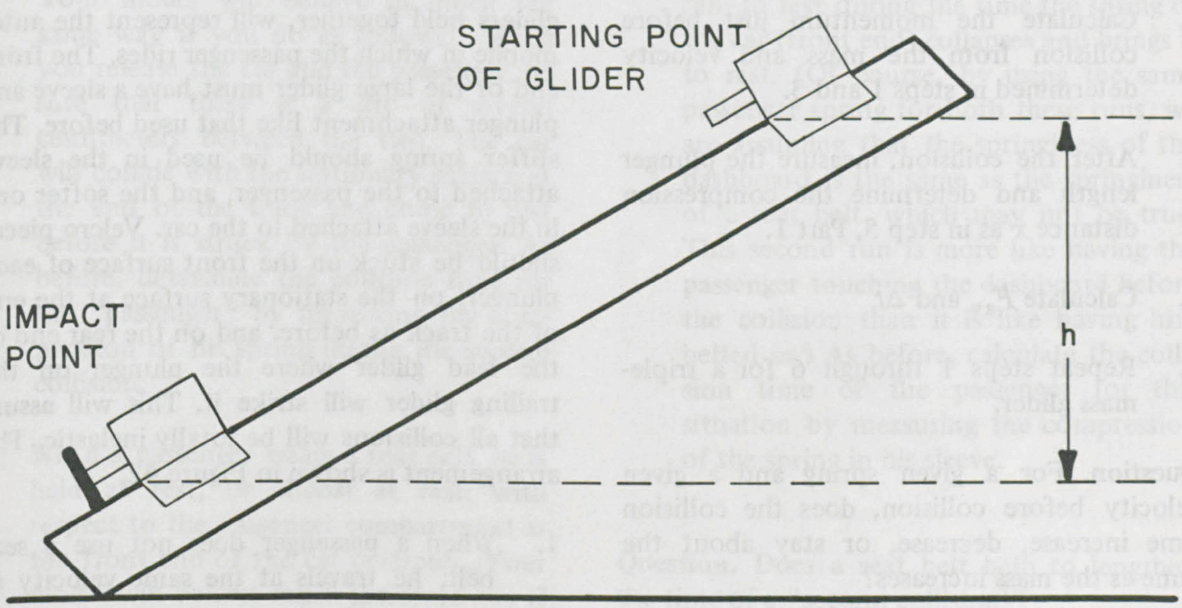

Figure 41.

Question. For a given spring and given glider, does the time of collision increase, decrease, or stay about the same as the velocity increases?

PART 2

In this part of the experiment, you will always release the glider from the same point on the track, using the full length of the track, but the mass of the glider will be changed each time. When several quantities affecting an experiment may be varied, it is instructive to vary one at a time. The data recorded for step 9 of Part 1 will serve as that needed for the first run of Part 2.

1. Attach another glider to the one equipped with the plunger and sleeve. Determine the mass of the combination and record its value.
2. Record the length of the plunger sticking out of the sleeve, as before.
3. Release this two-glider combination from a point on the track that allows movement along the full track ending in an inelastic collision with the Velcro at the end of the track. Calculate the velocity of the combination just before collision.
4. Calculate the momentum just before collision from the mass and velocity determined in steps 1 and 3.
5. After the collision, measure the plunger length and determine the compression distance x as in step 5, Part 1.
6. Calculate F_{av} and Δt.
7. Repeat steps 1 through 6 for a triple-mass glider.

Question. For a given spring and a given velocity before collision, does the collision time increase, decrease, or stay about the same as the mass increases?

PART 3

In this part of the experiment, repeat Part 1, or at least some runs of Part 1, using a "stiffer" spring (one with a larger spring constant k). Measure the value of k as before, then repeat steps 2 through 9 of Part 1.

Question. Does the collision time increase, decrease, or stay about the same as k increases?

PART 4

Repeat one or more runs as carried out in Part 1, but decrease the contact area of the Velcro surfaces. This area can be decreased easily, since the head of the plunger can be unscrewed and replaced with a smaller one.

Question. Does the collision time increase, decrease, or stay the same when the collision contact area increases?

PART 5

In this part of the experiment, you will try to determine if a seat belt helps to lengthen the time it takes to bring a "passenger" to rest during a collision. To do this, you will experiment with a "model." The modified glider used in Part 1 will represent a passenger. A more massive glider, or several gliders held together, will represent the automobile in which the passenger rides. The front end of the large glider must have a sleeve and plunger attachment like that used before. The stiffer spring should be used in the sleeve attached to the passenger, and the softer one in the sleeve attached to the car. Velcro pieces should be stuck on the front surface of each plunger, on the stationary surface at the end of the track as before, and on the rear end of the lead glider where the plunger on the trailing glider will strike it. This will assure that all collisions will be totally inelastic. The arrangement is shown in Figure 42.

1. When a passenger does not use a seat belt, he travels at the same velocity as

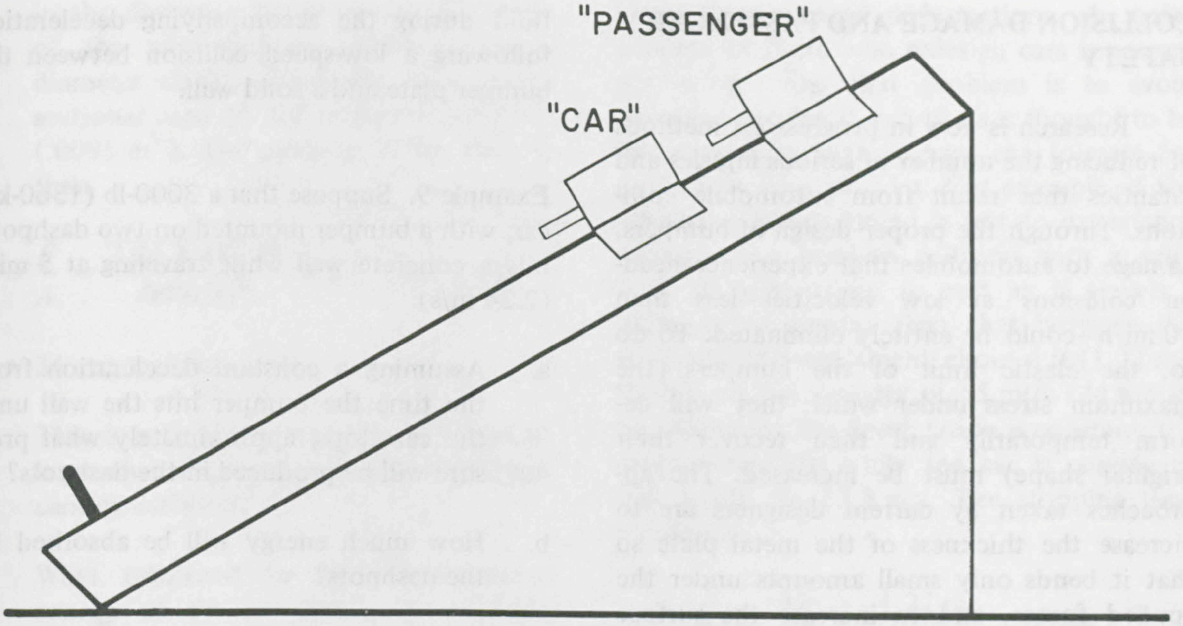

Figure 42.

the car until the "first collision" (car with outside object) occurs. To simulate this, release the passenger and the car simultaneously from elevated locations on the track so that they accelerate together down the track at the same rate. When the collision occurs, the car comes to rest as its front end crumples, but the passenger continues forward without slowing down until he suffers a "second collision" with the dashboard. Your model will behave in much the same way if you do as follows: When you release the car and the passenger, be sure that there is a gap of several centimeters between the two. The car will collide with the stationary surface at the end of the track and come to rest before it is struck by the passenger. As before, determine the collision time for the "passenger" by measuring the compression of his spring during the second collision.

2. When a passenger wears a seat belt, he is held at rest, or almost at rest, with respect to the passenger compartment as the front end of the car collapses. Your model will behave much the same way if you again release the car and passenger simultaneously. Be sure the car starts from the same point as before, but this time leave no gap between the two gliders. That is, the rear Velcro patch on the car should touch the Velcro patch on the passenger. However, the spring in the sleeve of the passenger must not be compressed at the time of release. When the car strikes the barrier, the spring (seat belt) of the passenger will bring him to rest during the time the spring of the car (front end) collapses and brings it to rest. (Of course, by using the same passenger spring for both these runs, we are assuming that the springiness of the dashboard is the same as the springiness of a seat belt, which may not be true. This second run is more like having the passenger touching the dashboard before the collision than it is like having him belted in.) As before, calculate the collision time of the passenger for this situation by measuring the compression of the spring in his sleeve.

Question. Does a seat belt help to lengthen the time of a "second collision"?

COLLISION DAMAGE AND PASSENGER SAFETY

Research is now in progress on methods of reducing the number of serious injuries and fatalities that result from automobile collisions. Through the proper design of bumpers, damage to automobiles that experience head-on collisions at low velocities–less than 10 mi/h–could be entirely eliminated. To do so, the elastic limit of the bumpers (the maximum stress under which they will deform temporarily and then recover their original shape) must be increased. The approaches taken by current designers are to increase the thickness of the metal plate so that it bends only small amounts under the applied forces, and to increase the surface area so that the forces are distributed over a larger area and the average stress is reduced.

However, even at 10 mi/h, an automobile experiences large forces if it is stopped quickly. Unless the entire body is made of very strong material–a practice that would increase the mass beyond reason–some part of the car must be capable of withstanding a fairly large compression without experiencing permanent deformation. One solution is to build fluid-filled "dashpots" between the metal bumper plate and the body of the car (see Figure 43). We can calculate the stress on the metal bumper and the pressure in the

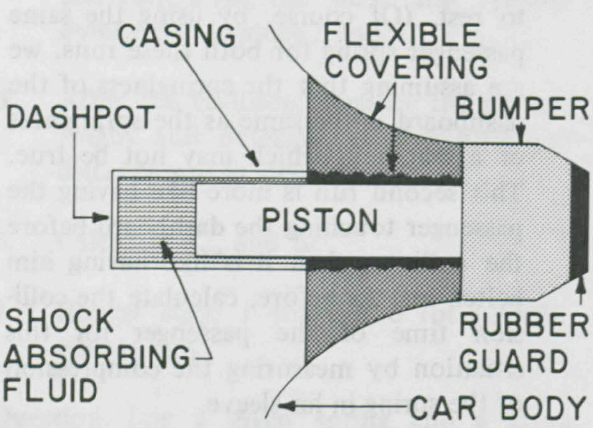

Figure 43. A dashpot-mounted bumper. When the bumper hits something, the piston is pushed to the left. Fluid is forced through small holes in the piston, and energy is absorbed gradually as a result.

fluid during the accompanying deceleration following a low-speed collision between the bumper plate and a solid wall.

Example 9. Suppose that a 3000-lb (1360-kg) car, with a bumper mounted on two dashpots, hits a concrete wall while traveling at 5 mi/h (2.24 m/s).

a. Assuming a constant deceleration from the time the bumper hits the wall until the car stops, approximately what pressure will be produced in the dashpots?

b. How much energy will be absorbed by the dashpots?

Solution.

a. The forward motion of the body of the car after the bumper hits cannot exceed some distance that is a reasonable length for the two dashpots. Let us take this distance (Δx) to be 6 in (0.152 m). The average velocity of the body of the car as it comes to rest under a constant force (constant deceleration) is half the initial velocity, or 1.12 m/s. The time of deceleration is then

$$\Delta t = \frac{\Delta x}{V_{\text{av}}} = \frac{0.152 \text{ m}}{1.12 \text{ m/s}} = 0.136 \text{ s}$$

Thus the deceleration of the car body is

$$a = \frac{\Delta V}{\Delta t} = \frac{2.24 \text{ m/s}}{0.136 \text{ s}} = 16.5 \text{ m/s}^2$$

The force required to produce this deceleration is

$$F = ma = 1360 \text{ kg} \times 16.5 \text{ m/s}^2$$

$$= 2.24 \times 10^4 \text{ N}$$

Assuming that the stresses in the steel bumper don't bend it out of shape, the force is transmitted directly to the fluid

in the dashpots. If we assume that each of the two dashpot cylinders has a diameter of 3 in, then the total cross-sectional area of the pistons is .098 ft^2 (.0091 m^2). The pressure in the fluid is then

$$P = \frac{F}{A} = \frac{2.24 \times 10^4 \text{ N}}{.0091 \text{ m}^2} = 2.5 \times 10^6 \text{ N/m}^2$$

$$\approx 24 \text{ atmospheres}$$

This is a large pressure, but it is not so large that a properly designed dashpot cannot withstand it.

b. What happened to the initial kinetic energy of the car during the collision described above? The amount was

$$K = (½)\, mV^2 = \frac{1360 \text{ kg}}{2} (2.24)^2 \text{ m}^2/\text{s}^2$$

$$= 3410 \text{ J}$$

To store this energy in two elastic springs that compress no more than 0.152 m, we would need springs with a spring constant of $k = 1.5 \times 10^5$ N/m. While it is possible to build large, elastic springs of this strength, there is a good reason not to use them for this purpose. If we did, then the car would rebound at its initial velocity, giving the passengers an impulse twice as large as the one needed to bring them to rest. Instead, we can cause the energy to end up as heat energy in the compressed fluid. If the fluid in the cylinders described above were water, the 3410 J (815 calories) of energy would raise its temperature only 0.6°C. The production of this heat energy need not be associated with any permanently deformed metal.

Passenger Safety

At greater velocities than that of the example above, no structure of reasonable mass has an elastic limit high enough to prevent permanent deformations. A major concern of those who redesign cars is passenger safety. The first problem is to avoid excessive decelerations; 40 *g*'s is thought to be the maximum that humans can tolerate for periods as long as 0.1 s. For example, if the passenger compartment is not to experience damage in a head-on collision with a solid wall, it must come to rest as it travels a distance no greater than that between the grille and the windshield, about 4 ft (1.22 m). With an initial velocity of 55 mi/h (24.6 m/s) and assuming the deceleration is constant, the average velocity while the car is coming to rest would be 12.3 m/s. The stopping time would be

$$\Delta t = \frac{\Delta x}{V_{\text{av}}} = \frac{1.22 \text{ m}}{12.3 \text{ m/s}} = 0.099 \text{ s}$$

The constant deceleration then is

$$a = \frac{\Delta V}{\Delta t} = \frac{24.6 \text{ m/s}}{0.099 \text{ s}} = 247 \text{ m/s}^2 \approx 25\, g\text{'s}$$

However, the force exerted by a collapsing metal structure is not really constant. It is small while the amount of compression is still small, and it becomes large when the deformation becomes large. If it behaves approximately as a spring behaves, then the deceleration would grow in proportion to the compression distance, and the maximum deceleration would be twice the average computed above. This would be somewhat excessive.

To improve this situation, designers propose to add cross braces to the front end of the car to increase the resisting forces as these structures first start to collapse. Research with prototype vehicles built in this way reveals that one can produce larger than average early decelerations for very short times. A prompt reduction in velocity means that the average velocity over the stopping time interval is smaller than it is for conventional cars. Thus, the time taken to cover the distance available is longer, and the average deceleration is smaller. See Figure 44 for graphs of the acceleration, velocity, and displacement of a standard car and a modified

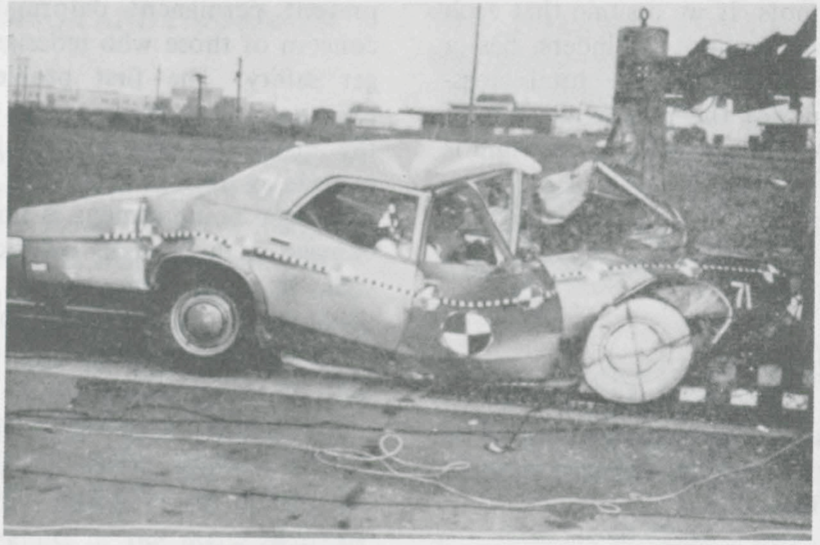

Figure 44A. A conventional car hits a solid post at 55 mi/h.

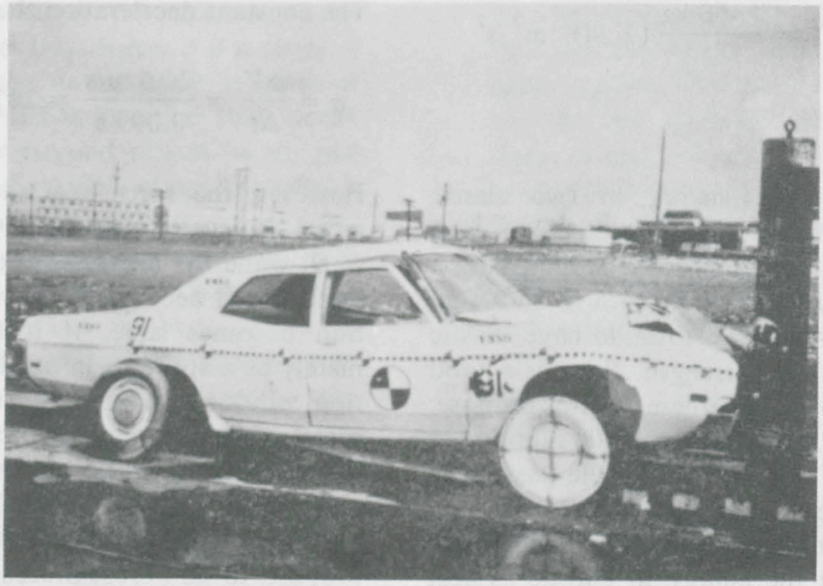

Figure 44B. A similar but modified car hitting the post at the same speed suffers far less damage.

car. Both traveled at 21.5 mi/h and Figure 44 shows the results of cross bracing of this type. In 44B, the modified car suffers far less damage to the passenger compartment and, presumably, to the passengers as well.

There are other problems and other possible solutions in keeping the passenger compartment intact as the structures in front of it are collapsing. Part of the solution is to take care that no structures that an auto is likely to collide with, including other cars, present sharp, strong objects that could penetrate the compartment like a sword. Another part of the solution is to strengthen the compartment at places where intrusions are likely to occur. The most dangerous projectile, and one that cannot be eliminated, is the motor of the car. Cars have been designed with extra strong and well-anchored *fire walls* between the engine and passenger compart-

ment which are slanted so that the passenger compartment will ride up over the motor during high-speed collisions. Not only does this keep the motor out of the driver's lap, but some of the energy is used up in lifting the car.

At this point you may wish to look again at the film loop showing real auto collisions with a post. In one sequence it shows a car riding up over its engine and thus decreasing the damage.

It is interesting to think about where the initial kinetic energy goes during a collision. Since the velocity before collision of the vehicle moving at 55 mi/h is 11 times that of a car moving at 5 mi/h, the kinetic energy of the same vehicle at 55 mi/h is 121 times the amount calculated at the lower velocity. This large amount of energy, 4.19×10^5 J (100,000 cal) for a 1360-kg car, will heat the metal structures that are permanently deformed and also any plastic materials, such as Urethane, that some designers propose to put between the bumper and the metal frame to absorb energy.

Statistics show that almost as many auto accidents are side collisions and roll-overs as head-on collisions. More important, the percentage of side collisions and roll-overs that result in serious injuries or fatalities is almost twice as large as the percentage of head-on collisions that have similarly disastrous results. This suggests that another area that needs attention is the strengthening of side panels and roofs. A lightweight but strong "honeycomb" material made from aluminum has been developed and promises to provide the added strength that is needed without adding greatly to the overall mass of the car.

Finally, we will comment on the controversy concerning the relative safety of high-mass and low-mass cars. When two cars of different mass collide, the impulse exerted on each, thus the change of momentum of each, is the same in magnitude. Therefore, the lower mass car experiences a greater change in velocity and hence a greater acceleration. These larger accelerations, to the extent that they are shared by the passengers, exert larger forces on the passengers, and thus injury is more likely. Since the distance from the driver's head to the windshield is less in a compact car than in a full-size car, there is less time before the "second collision" occurs. This means that devices like air-filled bags, which can be filled just barely fast enough to be helpful in standard-size cars, are probably useless in small cars. It appears that seat belts will continue to offer the only real hope of avoiding the injury-producing second collisions.

The fact that seat belts work so well may seem puzzling when you consider that the same momentum (of the passenger) must be eliminated whether the impulse is provided by the dashboard or the seat belt. However, as you have seen, the force that produces a given impulse is smaller if it is spread out over a longer time. The seat belt has more "give" than a dashboard, even a padded one, and thus your stopping time will be longer when you use a belt. In addition, an unbelted passenger does not hit the dashboard until the motion of the compartment has stopped, so his stopping distance is limited by the give of the dashboard. In contrast, a belted passenger is brought to rest as the front end of the car collapses, and this stopping distance is much greater. (This effect was exhibited in Part 5 of Experiment C-1.) Finally, injury depends more on local stress than on total force. An impulse caused by a corner of the dashboard may produce a disastrous stress on a small area of one's skull, whereas the same total force distributed over the entire area in contact with a waist belt and a shoulder harness may result in a non-damaging average pressure.

Another problem with compact cars is that small metal structures collapse under smaller forces than are required to collapse more massive structures. Thus, in a head-on collision with a large car where the forces on each car are the same, more damage is done to the small car.

However, when one considers a totally inelastic collision between a moving auto and an immovable wall, the advantage of the large car disappears. If two cars have the same velocity, the one with the lower mass has the

lower energy, and since the length of crushable structure in front of the passenger compartment is not proportionately shorter in a smaller car, there is a greater crushing distance available per unit of energy that must be absorbed in the smaller car. It therefore appears that the hazards of driving compact cars would disappear if all cars were the same size. In fact, if the modifications discussed were adopted by the manufacturers, overall safety would increase as the size of cars decreased if all were of uniform size.

SUMMARY

The center of mass of a system of masses can be calculated from a knowledge of the location and the size of each mass. The calculation amounts to finding a "weighted average" of all positions, where the weighting factor is the mass located at each position. (See the Appendix.)

Impulse is average force multiplied by the time the force acts ($F\Delta t$). The impulse on a body is equal to the change in the momentum of the body during the time the force acts:

$$F\Delta t = \Delta p$$

Until a limit, called the elastic limit, is reached, an object is called elastic. The amount of deformation (strain) it undergoes is proportional to the stress, which is force per unit area. Objects may be strong in the sense that it takes a large stress to produce a given strain, or they may be strong in the sense that the elastic limit is large and therefore permanent deformation is hard to produce.

In an inelastic collision, some of the kinetic energy of moving vehicles can be converted to rotational kinetic energy, elastic potential energy, or gravitational potential energy without contributing to permanent damage. The remainder is associated with permanent damage, and eventually appears in the form of heat energy.

The major threat to passenger safety is the "second collision." Any technique that can be used to lengthen the stopping time, such as the use of seat belts, reduces the magnitude of destructive forces on participants in collisions.

APPENDIX. Center of Mass

There is a way of finding the center of mass of a collection of masses from knowledge of where the masses are and how much mass is concentrated at each point. The center of mass is, in a sense, an average position of all the masses. However, in calculating this average, each position is *weighted* by the value of the mass at that location. That is, each position is multiplied by the mass located at that position as the average is taken.

The formula that expresses this weighted average—the position of the center of mass—is:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

The positions, $x_1, x_2, \ldots$, are those of the corresponding masses, as indicated in Figure A-1. If the mass is not distributed along a single straight line, then similar formulas must be used to find y_{cm} and z_{cm}, the location of the center of mass along the y and z axes.

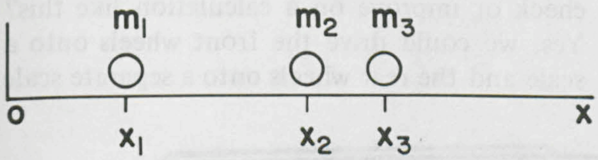

Figure A-1.

Although the equation may not remind you of the way you have calculated averages before, it is based on the same idea. Here, the values of x in the numerator are weighted by multiplying by mass, and the denominator is the total mass involved.

One simple example is a binary star system. A binary star system contains two stars which are held close together by their gravitational forces. They rotate around their center of mass. If the stars have the same mass, then you would expect that their center of mass might be halfway between them. Figure A-2 illustrates how we would apply Equation (10) to the computation of the center of mass. Their center of mass must lie on the line drawn between their centers. (Why?) To locate it, choose an origin and apply the preceding equation. The position of the center of mass is

$$x_c = \frac{mx_1 + m(x_1 + d)}{m + m}$$

$$= \frac{(2x_1 + d)}{2} = x_1 + \frac{d}{2}$$

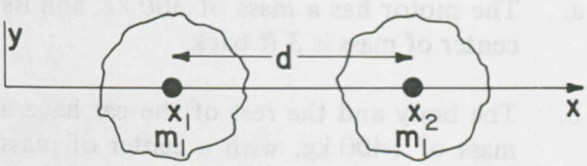

Figure A-2.

Since d is the distance between the stars, the center of mass is located halfway between them, as you probably guessed. What happens if one star is much more massive than the other? Suppose that $m_2 = 10\, m_1$. Then

$$x_c = \frac{m_1 x_1 + 10\, m_1 (x_1 + d)}{m_1 + 10\, m_1}$$

$$= \frac{m_1 (11\, x_1 + 10\, d)}{11\, m_1}$$

$$= x_1 + \frac{10}{11} d$$

Now the center of mass is very close to the more massive star.

In the system used in Experiment A-2 (a meter stick with a cylindrical mass taped to the end), both objects have mass distributed throughout a region with dimensions of several centimeters or more. However, from symmetry we know that the center of mass of the stick is at the 50-cm mark, and the center of mass of the cylinder is at its geometric center. To find the center of mass of the

entire system, it is correct to use the previous formula where m_1 is the mass of the meter stick, $x_1 = 50$ cm, m_2 is the mass of the cylinder, and x_2 is the location of the cylinder on the stick. The center of mass will be between the 50-cm mark and the taped-on mass.

Example. A family is going on a camping trip in their station wagon. The wagon is 18 ft long and, empty, it has a mass of 1700 kg. All of the following measurements of position are from the front bumper.

a. The motor has a mass of 300 kg, and its center of mass is 3 ft back.

b. The body and the rest of the car have a mass of 1400 kg, with a center of mass 10 ft back.

c. The parents in the front seat total 120 kg, 8 ft back.

d. The children in the back seat are 11 ft back and have a total mass of 80 kg.

e. The rear deck is piled with 200 kg of camping gear whose center of mass is at 15 ft.

All of this is indicated in Figure A-3. Where is the center of mass of the whole mess?

Solution. The units are mixed metric and English, but that is no problem because the kg will cancel in numerator and denominator and leave the answer in ft.

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3 + m_4 + m_5} + \frac{m_4 x_4 + m_5 x_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

$$= \frac{300 \text{ kg} \times 3 \text{ ft} + 1400 \text{ kg} \times 10 \text{ ft}}{2100 \text{ kg}} + \frac{120 \text{ kg} \times 8 \text{ ft} + 80 \text{ kg} \times 11 \text{ ft}}{2100 \text{ kg}} + \frac{200 \text{ kg} \times 15 \text{ ft}}{2100 \text{ kg}}$$

$$= 9.45 \text{ ft}$$

Is there an experimental way we can check or improve on a calculation like this? Yes, we could drive the front wheels onto a scale and the rear wheels onto a separate scale

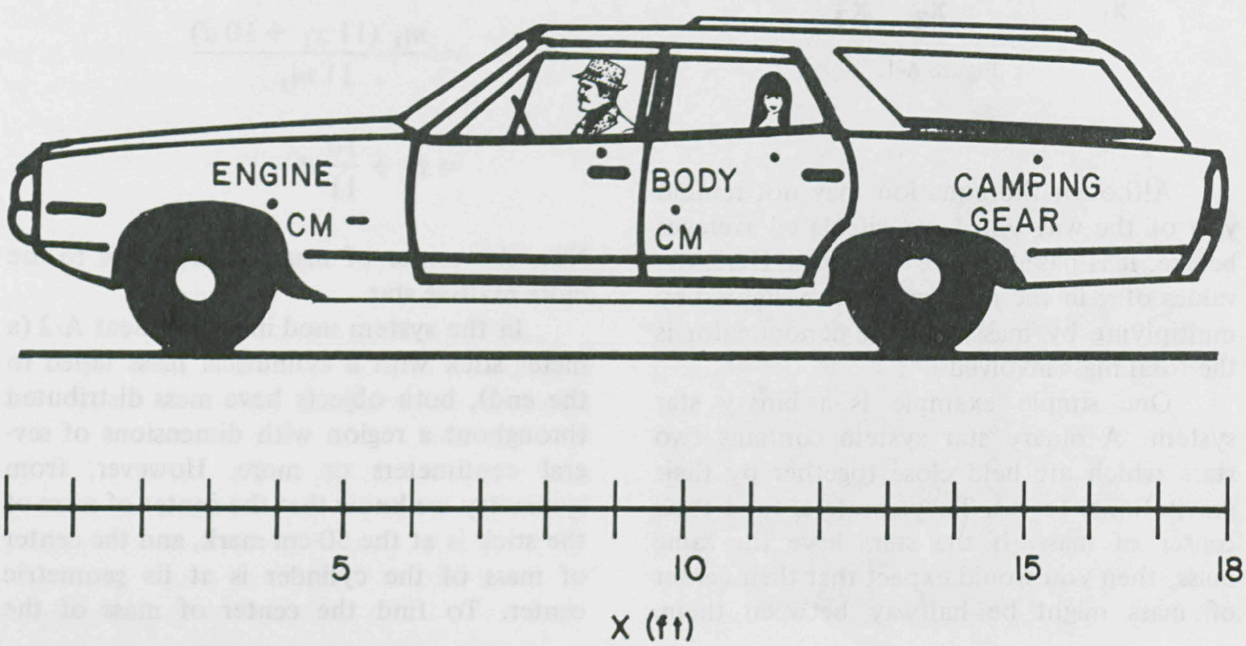

Figure A-3.

and read the force that each exerts on the car. Then we could proceed as in Experiment A-3.

Problem A-1. How much weight does each tire of the wagon of the preceding example support on a level road? (1 kg weighs 2.2 lb.)

Problem A-2. The wagon of the example is 4.8 ft high, and the center of mass when loaded is 2.0 ft above the ground. How much would it raise the center of mass to put a 200-kg load on the roof rack, with its center of mass 5.0 ft above the ground?

DATA PAGE

EXPERIMENT B-2. Simulated Collisions

PART 1. Moving Car and Stationary Truck

TABLE I.

	Before Collision			After Collision	Difference Between Before and After
	Car	Truck	Total		
Velocity					
Momentum					
Kinetic Energy					

PART 2. Moving Car Overtakes Moving Truck

TABLE II.

	Before Collision			After Collision	Difference Between Before and After
	Car	Truck	Total		
Velocity					
Momentum					
Kinetic Energy					

PART 3. Car and Truck Approaching Each Other

TABLE III.

	Before Collision			After Collision	Difference Between Before and After
	Car	Truck	Total		
Velocity					
Momentum					
Kinetic Energy					

PART 4 (OPTIONAL). Collision 1 Viewed from the Center of Mass

TABLE IV.

	Before Collision			After Collision	Difference Between Before and After
	Car	Truck	Total		
Velocity					
Momentum					
Kinetic Energy					

TABLE IVb.

	Before Collision			After Collision	Difference Between Before and After
	Car	Truck	Total		
Velocity					
Momentum					
Kinetic Energy					

PART 5 (OPTIONAL). Collision in Two Dimensions

TABLE V.

	Before Collision			After Collision	Difference Between Before and After
	Car	Truck	Total		
Velocity					
Momentum					
Kinetic Energy					

0-07-001726-3